1. Introduction. Panel flutter phenomenon is a serious problem well known in supersonic aviation. Let us consider a skin panel of a flight vehicle (airplane, rocket, etc, see Fig. 1 for example). If the flight Mach number is low, the panel is stable, however at some critical Mach number, \( M_{cr} \), the panel becomes unstable and flutter occurs. Amplitude of the vibrations can be high and can result in instantaneous or fatigue destruction of the panel or hydraulic drives and other structures attached to the panel.

The phenomenon of panel flutter was first observed during the 1940s and has since had a rich history. Theoretical solution of the problem consists of an eigenvalue solution of coupled panel-flow equation (for simplicity we consider 2-D problem):

\[
D \frac{\partial^4 w}{\partial x^4} - \frac{\partial^2 w}{\partial t^2} + p(x,t) = 0,
\]

here \( D \) is dimensionless plate stiffness, \( w \) is the plate deflections, and \( p(x,t) \) is the gas pressure (Fig. 2). Let us assume that the plate deflection is harmonic:

\[
w(x,t) = W(x)e^{-i\omega t},
\]

then the pressure obtained from theory of potential gas flow has the form

\[
P(x) = \frac{\mu M}{\sqrt{M^2 - 1}} \left( i\omega W(x) + M \frac{\partial W(x)}{\partial x} \right) + \frac{\mu \omega}{(M^2 - 1)^{2/3}} \int_{-L/2}^{L/2} \left( i\omega W(\xi) + M \frac{\partial W(\xi)}{\partial \xi} \right) \times
\]

\[
\times \exp \left( \frac{iM\omega(x - \xi)}{M^2 - 1} \right) \left( iJ_0 \left( \frac{-\omega(x - \xi)}{M^2 - 1} \right) + MJ_1 \left( \frac{-\omega(x - \xi)}{M^2 - 1} \right) \right) d\xi
\]

In 1956, a simple approximation of the gas pressure at high Mach numbers, known as piston theory, was derived. This theory neglects integral term in (1.2), reduces the problem of panel flutter to a simple partial-differential equation. Since this simple mathematical formulation of the problem was derived, a lot of studies, analytical, numerical and experimental, have been conducted at high Mach numbers.

However, at low \( M>1 \) the problem has not been studied properly. The reason of this is that such a study requires solution of full system (1.1), (1.2), which is very difficult because of sophisticated integral term in (1.2). In a few studies [1-5] where...
low supersonic Mach numbers were considered, it was noticed that at low supersonic Mach numbers a new type of panel flutter, called “single mode flutter”, can occur. As it occurs at lower flight speeds than classical flutter, it may be more dangerous for flight vehicles, but no detailed investigations have been conducted.

Studies of single mode panel flutter conducted in Lomonosov Moscow State University during last years are in a whole range of investigations: analytical, numerical and experimental. They are considered hereunder in series.

2. Analytical studies of single mode panel flutter. During last several years a great progress in studies of single mode flutter was achieved [6-10]. The main feature is that the method of investigations used [11, 12] is absolutely non-typical and has never been applied in aeroelasticity. The main idea is to consider eigenmode of the plate as a superposition of plane waves running along the plate. Then, studying influence of the flow on each wave (Fig. 3), we can calculate summary influence of the flow on the eigenmode itself.

Figure 3. Gas pressure distribution along elastic wave running along the plate. Flow speed is subsonic (a), supersonic (b) with respect to the wave.

The result [6-8] is as follows: each mode with dimensionless frequency $\omega$ lying in the range $\omega^{*}<\omega<\omega^{**}$, where

$$\omega^{*} = (M - 1)\sqrt{((M - 1)^2 - M_w^2)}/D,$$

$$\omega^{**} = \sqrt{(M^2 + 1 - \sqrt{4M^2 + 1})(M^2 + 1 - \sqrt{4M^2 + 1 - M_w^2})}/D,$$

is unstable. Other modes are stable with respect to single model flutter. If at least one natural frequency lies in above-mentioned range, then the plate is unstable. Thus, single mode flutter criterion is derived.

3. Numerical study of single mode flutter. Closed-form solution [6-8] is derived in assumption that the dimensionless plate length $L>>1$. In order to make sure that this condition is satisfied for real structures, numerical study of the problem (1.1), (1.2) has been conducted through Bubnov-Galerkin method. Representing solution as a superposition of the plate natural modes,

$$W(x) = \sum_{n=1}^{N} C_n W_n(x), \quad W_n(x) = \sin\left(\frac{n\pi x}{L}\right),$$

substituting it in the governing equations and conducting Bubnov-Galerkin procedure, we obtain frequency equation for the plate in a gas flow.

Calculations are conducted for a steel plate in air flow [9]. Let us fix the plate length $L = 300$ and change Mach number, $M$. In Fig. 4 shown are trajectories of eigenfrequencies for $1.05<M<1.5$ (solid lines are solutions of the full system (1.1), (1.2), dashed lines are solutions of piston theory model.

One can see that for each plate eigenmode there is a region of single-mode instability (where $\text{Im}\, \omega > 0$), as was predicted by the closed-form solution. Detailed comparison of flutter boundaries in parameter space shows that quantitatively analytical solution also works very well.
Figure 4. Trajectories of the first four eigenfrequencies in complex $\omega$ plane for $M$ changed in the range $1.05 < M < 1.5$. 1, 2 are frequencies at $M=1.05, 1.5$; 3 are natural frequencies of the plate in vacuum.

4. Experimental flutter studies. Single mode flutter is a phenomenon initially discovered theoretically and never observed in tests. In order to make sure that this phenomenon occurs and can be dangerous, set of experiments was conducted in supersonic wind tunnel of Institute of Mechanics [10]. The model is a steel plate, 540x300x1 mm size, welded to a rigid frame installed on the wind tunnel wall.

![Figure 5](image)

Figure 5. Sketch of the test (a) and model installed into the tunnel (b). (1) plate, (2) frame, (3) cavity, (4) wind tunnel walls.

During the test, plate vibrations, air pressure pulsations and wind tunnel vibrations were controlled through 12 strain gauges, pressure gauge and vibration gauge. During data postprocessing, spectral analysis of each gauge was conducted. Detection of the plate vibration type is possible through analysing of spectrum data.

![Figure 6](image)

Figure 6. Dynamic strain amplitude vs Mach number (a). Plate strain vs time at $M=1.147$ (stability) and 1.298 (flutter) (b).

In Fig. 6 (a) shown is the plate vibration amplitude versus Mach number of the flow. At $M \approx 1.2$ one can see sharp growth of the amplitude. Analysis of strain,
pressure and vibro gauge spectra clearly indicates that this vibration type is single mode flutter, Fig. 6 (b). Theoretical prediction based on analytical results [6-8] gives flutter boundary at $M=1.17$, which is very close to experimentally observed value. Modes which are unstable in flutter also coincide with theoretical results.

Thus, for the first time single mode panel flutter was observed in experiment, and experimental data are very close to the theoretical results.

5. Conclusions and future plans. Panel flutter studies conducted in Lomonosov Moscow State University deal with a new phenomenon, which has not been studied previously – single mode panel flutter. Analytical theory has been derived and detailed numerical studies have been conducted. Experiments conducted proved existence of single mode flutter and accuracy and exactness of theoretical predictions.

Among future plans there are direct numerical simulation of the plate oscillations in air flow through direct coupling of elastic and aerodynamic codes, investigation of boundary layer influence and further experimental studies.

Acknowledgments. This work was supported by the Russian Foundation for Basic Research (projects no. 08-01-00618 and 10-01-00256) and the Grants of President of Russian Federation (MK-2313.2009.1 and NSh-4810.2010.1)

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