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Flutter of Infinite Elastic Plates in the Boundary-Layer Flow at Finite Reynolds Numbers

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Abstract—The stability of an infinite elastic plate in supersonic gas flow is investigated taking into account the presence of the boundary layer formed on the plate surface. The effect of viscous and temperature disturbances of the boundary layer on the behavior of traveling waves is studied at large but finite Reynolds numbers. It is shown that in the case of the small boundary layer thickness viscosity can have both stabilizing and destabilizing effect depending on the phase velocity of disturbance propagation.

Key words: boundary layer, panel flutter, plate flutter, hydrodynamic instability.

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The panel flutter is the aeroelastic instability of aircraft skin panels which can lead to their destruction. Earlier, it was shown that along with the well-known flutter of the coupled-mode type, studied in detail [1], which arises at high supersonic speeds, a single-mode flutter [2] excited at trans- and low supersonic speeds can also develop. The single-mode flutter cannot be revealed and investigated by means of the piston theory usually used in supersonic aeroelasticity. These results were confirmed numerically and experimentally [3].

In many previous studies on panel flutter the gas viscosity was neglected and flow was assumed to be homogeneous. However, in theoretical [4–10] and experimental [11, 12] studies it was shown that flutter can be reduced or completely suppressed in the presence of the boundary layer. This result was obtained in analysis of a particular boundary layer profile, namely, that corresponding to the zero-pressure-gradient turbulent boundary layer on a flat plate. However, qualitatively different boundary layer profiles can develop during the flight in various parts of the surface depending on the flow conditions.

In [13–15] the influence of the boundary layer of an arbitrary form on panel flutter was investigated analytically in the approximation of the inviscid shear layer. Its influence on the coupled-mode flutter is independent of the boundary layer profile. At the same time, in the case of long waves the influence of the boundary layer on the single-mode flutter depends on the type of the boundary layer profile, namely, the influence is different for the generalized convex profile and the profile with the generalized inflection point. In [16] the results of [13–15] were generalized to arbitrary wavelengths by solving the Rayleigh equation and the dispersion relation numerically. Comparison with the analytic solution [13, 15] was carried out and the wavelength range on which these solutions coincide was found.

So far the investigations of the influence of the boundary layer of the general form on flutter were carried out in the inviscid formulation (Reynolds number $R = \infty$) and the effect of viscous and temperature perturbations of the boundary layer was not studied. In the present paper the effect of the boundary layer on the behavior of traveling waves of an elastic plate is investigated with regard to viscosity at large but finite Reynolds numbers. It is well known that waves traveling in the infinite plate largely determine the boundaries of the single-mode flutter [17]. Thus, the present paper is the first stage in the study of the influence of the viscous boundary layer perturbations on the flutter of finite plates.

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(a) (b)

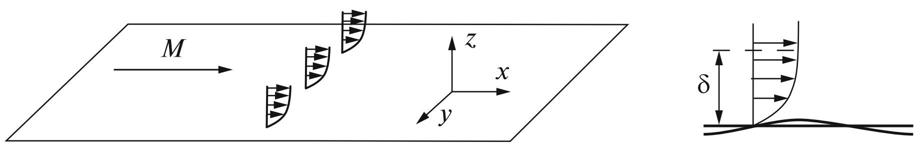

Fig. 1. Infinite plate in supersonic gas flow (a); boundary layer on the surface of an elastic plate (b).

1. FORMULATION OF THE PROBLEM

We will investigate the stability of the following system: an elastic plate exposed on one side to a plane-parallel supersonic flow of a perfect viscous gas. The plate is considered to have the shape of infinite plane and on its surface there is a boundary layer with the velocity and temperature profiles assumed to be given (Figs. 1,a and 1,b). Small perturbations in the form of traveling waves are imposed on the system, namely, the plate has the deflection $w(x,t) = e^{i\alpha(x-ct)}$ and the flow parameters have the perturbations $\Psi_d(x,z,t) = \Psi(z)e^{i\alpha(x-ct)}$ in the Cartesian coordinate system x, y, z. Thus, the flow parameters have the form $\Psi_t(x,z,t) = \Psi^0(z) + \Psi(z)e^{i\alpha(x-ct)}$, where $\Psi^0(z)$ are functions which describe the undisturbed steady-state flow. The problem is solved in the plane formulation and the perturbations independent of the coordinate y are considered. Flow is assumed to be laminar and be also homogeneous when $z > \delta$, where δ is the boundary layer thickness.

Initially, we will consider the results of previous studies in which the inviscid approximation is investigated and the perturbation growth rates as functions of the boundary layer thickness are obtained for various wavelengths.

2. RESULTS OF INVESTIGATIONS IN THE INVISCID APPROXIMATION

In [13–16] the inviscid approximation of the problem of influence of the boundary layer on plate flutter was considered for supersonic gas flow, i.e., for the Reynolds number $\mathrm{R} = \infty$.

It was proved that the waves with the phase velocities $\mathrm{Re}c > \mathrm{M}$ or $\mathrm{Re}c < 0$, where M is the Mach number, are neutral or damped. Consequently, only the waves with the phase velocities $0 < \mathrm{Re}c < \mathrm{M}$ can lead to instability.

The boundary layer is called generalized convex if the condition $(u'/T)' < 0$ is satisfied for $z \in [0;\delta)$, where u and T are the velocity and temperature profiles of undisturbed flow and δ is the boundary layer thickness (here, prime denotes differentiation with respect to z). In [13, 15, 16] for the profiles of this type it was proved the following:

1. If the wave is growing at $\delta = 0$ (i.e., when $0 < \mathrm{Re}c < \mathrm{M} - 1$), then it remains also growing for $\delta \neq 0$. However, the growth rate decreases monotonically and tends to zero when δ increases.

2. If the wave is neutral at $\delta = 0$ (i.e., when $\mathrm{M} - 1 < \mathrm{Re}c < \mathrm{M}$), then for $\delta \neq 0$ it starts to grow. When δ increases, the perturbation growth rate initially increases, reaching a maximum at certain $\delta = \delta_m$, and then decreases and tends monotonically to zero.

The influence of the boundary layer was also investigated in the presence of a single generalized inflection point z_i at which $(u'/T)' = 0$. In subsonic flows such profiles are unstable since the existence of the generalized inflection point is necessary and sufficient for instability of subsonic perturbations in the inviscid approximation [18]. However, in supersonic flow there exist such profiles that the generalized inflection point is located in the supersonic part of the boundary layer, i.e., in the region in which the flow is supersonic with respect to main flow ($u(z_i) < \mathrm{M} - 1$). On the one hand, this means that the subsonic perturbations are damped since their stability criterion

$$\left(\frac{u'(z)}{T(z)}\right)' < 0, \quad z > z_s, \tag{2.1}$$

where $u(z_s) = \mathrm{M} - 1$, is satisfied. On the other hand, the supersonic perturbations can also be damped since for them the stability criterion [15] is not related to existence of the generalized inflection point. Consequently, such profiles can be stable and, therefore, can exist in real flows.

The boundary layers of such type affect the traveling wave as follows:

1. If the wave is growing at $\delta = 0$ (i.e., when $0 < \mathrm{Re}c < \mathrm{M} - 1$), then it remains growing for $0 < \delta < \delta_1$, the perturbation growth rate becoming greater than that in homogeneous flow. For the thicker boundary layers in which $\delta_1 < \delta < \delta_2$ the wave is growing but the perturbation growth rate is smaller than that in homogeneous flow. Finally, in the thick boundary layers in which $\delta > \delta_2$, the wave becomes damped.

2. If the wave is neutral at $\delta = 0$ (i.e., when $\mathrm{M} - 1 < \mathrm{Re}c < \mathrm{M}$), then for $\delta \neq 0$ it begins to grow. The behavior of such waves is similar to the case of the generalized convex profiles.

3. BOUNDARY LAYER PERTURBATIONS WITH REGARD TO VISCOSITY

3.1. System of Equations for Perturbations

In what follows, the Reynolds number will be assumed to be large but finite.

We will consider the linearized nondimensionalized system of gas dynamic equations for perturbations [18] in the form $dz_i/dz = \sum_{j=1}^{6} A_{ij} z_j$, where A_{ij} are known functions of z, and $z_j, j = 1, \ldots, 6$ are known functions for which we take

$$z_1 = f; \quad z_2 = f'; \quad z_3 = \varphi; \quad z_4 = \frac{\pi}{\mathrm{M}^2}; \quad z_5 = \theta; \quad z_6 = \theta',$$

where f and θ are perturbations of the horizontal velocity component and the temperature divided by the velocity u^*_∞ and the temperature T^*_∞, respectively. Here, the subscript ∞ denotes the values of the quantities in outer homogeneous flow ($z > 1$). The function φ characterizes the perturbation of the vertical velocity component divided by u^*_∞ and is related to it as follows: $v^*_z = u^*_\infty \alpha \varphi(z) e^{i\alpha(x-ct)}$, where α is the wavenumber. The coordinate z is nondimensionalized by the boundary layer thickness δ^*. The quantity π is the pressure perturbation divided by $\rho^*_\infty u^{*2}_\infty/(\gamma \mathrm{M}^2)$, where γ is the specific heat ratio. Since in the theory of boundary layer the free-stream pressure p is assumed to be independent of z, we will assume that $p(z) = \mathrm{const}$.

We will assume that there are no body forces and the dynamic viscosity perturbation depends on the temperature perturbation as follows: $m_1 = \theta(d\mu_{dyn}/dT)$, where μ_{dyn} is the dynamic viscosity divided by its value far from the plate. For the sake of simplicity, in what follows we will assume that the dynamic viscosity is constant and, respectively, $m_1 = 0$ (using considerably more cumbersome algebra, it can be proved that the same results, which will be obtained below, can be reached without these simplifications). Then the linearized nondimensionalized system of gas dynamic equations takes the form:

$$\begin{gathered}
z_1' = z_2, \\
z_2' = \frac{\alpha \mathrm{R}}{\nu\rho}\left(\rho(i(u-c)z_1 + u'z_3) + \frac{i}{\gamma}z_4\right) + O(1), \\
z_3' = -iz_1 - \frac{\rho'}{\rho}z_3 - i(u-c)\left(\frac{z_4\mathrm{M}^2}{p} - \frac{z_5}{T}\right), \\
z_4' = \left(1 + \frac{1}{\mathrm{R}}O(1)\right)^{-1}\left(-\gamma\alpha^2\rho i(u-c)z_3 + \frac{1}{\mathrm{R}}O(1)\right), \\
z_5' = z_6, \\
z_6' = \frac{\alpha \mathrm{R}\mathrm{Pr}}{\gamma\nu\rho}\left(\gamma\rho(T'z_3 + i(u-c)z_5) - i(\gamma-1)(u-c)z_4\mathrm{M}^2\right) \\
\qquad - 2\mathrm{Pr}(\gamma-1)\mathrm{M}^2u'(z_2 + i\alpha^2 z_3) + \alpha^2 z_5,
\end{gathered} \tag{3.1}$$

where ρ is the free-stream density nondimensionalized by ρ^*_∞, ν is the kinematic gas viscosity nondimensionalized by ν^*_∞, Pr is the Prandtl number, and $O(1)$ denotes functions of z_j of the order of unity as $\mathrm{R} \to \infty$. The Reynolds number is defined as follows: $\mathrm{R} = u^*_\infty \delta/\nu^*_\infty$.

The general solution of the system (3.1) represents a combination of six linearly independent particular solutions. As $\mathrm{R} \to \infty$, the solutions of (3.1) can be approximated by means of two regular solutions transformed into solutions of the Rayleigh equation in the limit of $\mathrm{R} = \infty$ and four solutions of the WKB-type [18] which have the asymptotic form $z_i(z) = f_i(z)\exp(g_0(z)\sqrt{\alpha R})$.

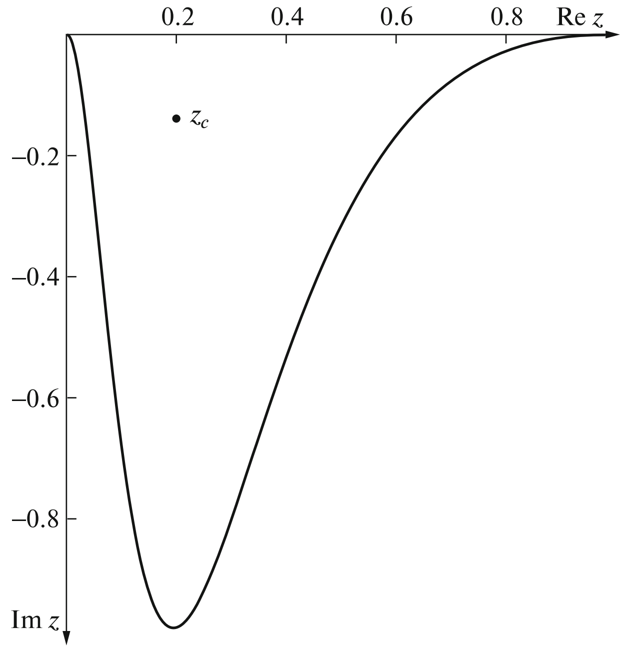

Fig. 2. Path of integration of the Rayleigh equation for the velocity profile $u(z) = \mathrm{M}\sin(\pi z/2)$; $\mathrm{M} = 1.6$, and $c = 0.5 - 0.33i$. The critical point $z_c \approx 0.20 - 0.14i$, $\delta = 1$.

3.2. Regular Solutions

Using the linearized nondimensionalized gas dynamic equations (3.1), in the inviscid approximation ($\mathrm{R} = \infty$) the following second-order ordinary differential equation for the vertical component of the perturbation velocity (compressible Rayleigh equation) can be obtained [18]

$$\frac{d}{dz}\left(\frac{(u-c)\varphi' - u'\varphi}{T - \mathrm{M}^2(u-c)^2}\right) - \frac{\alpha^2}{T}(u-c)\varphi = 0. \tag{3.2}$$

The pressure perturbation can be expressed in terms of the perturbation of the vertical velocity component by the formula

$$\pi = -i\gamma\mathrm{M}^2\left(\frac{(u-c)\varphi' - u'\varphi}{T - \mathrm{M}^2(u-c)^2}\right). \tag{3.3}$$

The Rayleigh equation has two singularities [18]. The first singularity located at the point at which $T(z) - \mathrm{M}^2(u(z) - c)^2 = 0$ is removable. The second singularity, located at the critical point z_c, where $u(z_c) = c$, represents the singularity of the logarithmic type and leads to singularity of the solution. This singularity cannot be removed within the framework of the inviscid case. Since there is a logarithmic singularity at the point z_c, the solution obtained by integration along the path, which passes from above around the critical point in the complex plane, in the general case is not equal to the solution obtained by integration along the path tracing from below around z_c. The solutions, which are the limit of the solutions of the viscous system as $\mathrm{R} \to \infty$, can be obtained when integration is carried out along a contour in the complex plane z with tracing from below around the critical point (Lin tracing rule) [18–20] (Fig. 2). Consequently, in solving the Rayleigh equation, integration must be carried out along a smooth curve passing from below around the critical point. In particular, in the case $\mathrm{Im}(z_c) > 0$ integration can be implemented along the real z-axis.

We will now consider the boundary conditions for the Rayleigh equation. We impose the impenetrability condition at the vibrating plate on the surface $z = 0$. The second condition is imposed at the outer boundary layer edge $z = 1$. Since flow is homogeneous at $z > 1$, the Rayleigh equation (3.2) reduces to the equation with constant coefficients and has the solution $v(z) = Ce^{-\beta z}$, where $\beta = \alpha\sqrt{1 - \mathrm{M}^2(1-c)^2}$. In this case such a branch of the root must be taken from the perturbation radiation condition as $z \to +\infty$ that $\mathrm{Re}\beta > 0$ when $\mathrm{Im}\omega \gg 1$. Outside the boundary layer this exponential solution must be matched with the solution inside the boundary layer. This is the second boundary condition. Thus, the boundary conditions take the form:

$$\varphi = -ic \quad (z = 0), \quad \frac{d\varphi}{dz} + \beta\varphi = 0 \quad (z = 1). \tag{3.4}$$

3.3. WKB Solutions

For calculation of the first terms of expansion of the WKB solutions in $\varepsilon = 1/\sqrt{\alpha \mathrm{R}}$ the unknown functions z_i are given in the form:

$$z_i = (f_{i0}(z) + f_{i1}(z)\varepsilon + f_{i2}(z)\varepsilon^2 + ...) \exp\left(\frac{g_0(z)}{\varepsilon}\right).$$

Substituting z_i in Eqs. (3.1), we obtain the following two types of the WKB solutions. The first solution which takes the form:

$$f_{10} = \mathrm{const}_1 (u - c)^{-\frac{3}{4}} \left(\frac{i}{\nu}\right)^{-\frac{1}{4}},$$
$$f_{30} = f_{40} = f_{50} = 0,$$
$$f_{31} = -i\mathrm{const}_1 (u - c)^{-\frac{5}{4}} \left(\frac{i}{\nu}\right)^{-\frac{3}{4}},$$
$$f_{41} = 0,$$
$$g_0 = \int\limits_{z^*}^{z} \sqrt{\frac{i}{\nu}(u - c)}dz \tag{3.5}$$

is called viscous solution. The second solution

$$f_{10} = f_{30} = f_{40} = 0,$$
$$f_{41} = 0,$$
$$f_{50} = \mathrm{const}_2 T^{\frac{1}{2}} (u - c)^{-\frac{1}{4}} \left(\frac{i\mathrm{Pr}}{\nu}\right)^{\frac{1}{2}},$$
$$g_0 = \int\limits_{z^*}^{z} \sqrt{\frac{i\mathrm{Pr}}{\nu}(u - c)}dz \tag{3.6}$$

is called temperature solution.

The viscous and temperature solutions contain pairs of the solutions which differ by choosing the branch in g_0.

4. FIRST APPROXIMATION IN $(\sqrt{\alpha \mathrm{R}})^{-1}$ FOR THE PRESSURE PERTURBATION

In the general case, if we do not consider initially the boundary conditions, then the solution of the system of equations (3.1) will consist of six particular linearly independent solutions, two of them are regular solutions, two are viscous solutions, and two are temperature solutions [18]. In order to satisfy the radiation condition at infinity, it is necessary to take the corresponding branches of the roots in g_0 in the viscous and temperature solutions (other branches correspond to the solutions growing exponentially as $z \to \infty$ and do not satisfy the radiation condition). Accordingly, a single solution of each type remains from two viscous and two temperature solutions. The linear combination which satisfies the radiation condition at infinity can be composed from two linearly independent regular solutions. Thus, the solution of the system of equations (3.1), which satisfies the radiation condition, represents the sum of three linearly independent solutions (regular, viscous, and temperature solutions)

$$f(z) = c_1 f_r(z) + c_2 f_v(z) + c_3 f_t(z),$$

$$\varphi(z) = c_1\varphi_r(z) + c_2\varphi_v(z) + c_3\varphi_t(z),$$
$$\theta(z) = c_1\theta_r(z) + c_2\theta_v(z) + c_3\theta_t(z). \tag{4.1}$$

Similarly for other unknowns, including the pressure perturbation

$$\pi(z) = c_1\pi_r(z) + c_2\pi_v(z) + c_3\pi_t(z), \tag{4.2}$$

where the subscripts r, v, and t denote the regular, viscous, and temperature solutions, respectively.

These solutions can be represented by the following expansions in ε:

$$f_r = f_{inv} + f_r^2\varepsilon^2 + \ldots;$$
$$f_{v,t} = (f_{v,t}^0 + f_{v,t}^1\varepsilon + f_{v,t}^2\varepsilon^2 + \ldots)\exp\left(\frac{g_{0(v,t)}}{\varepsilon}\right), \tag{4.3}$$

where the subscript "inv" denotes the inviscid solution at $\varepsilon = 0$, i.e., the solution of the Rayleigh equation.

In section 3.3 it was found that $f_{40} = f_{41} = 0$ for both viscous and temperature solutions. Moreover, since π_r is the regular solution of the system (3.1) in which ε enters only in the form ε^2, then

$$\pi_r(z) = \pi_{inv}(z) + O(\varepsilon^2).$$

From this fact it follows that $\pi(z)$ (4.2) can be written in the form:

$$\pi(z) = c_1(\varepsilon)\pi_{inv}(z) + O(\varepsilon^2). \tag{4.4}$$

In view of this fact, for finding the first approximation in ε for the pressure perturbation we need to calculate c_1, since only this quantity in (4.4) has the linear term in the expansion in ε.

For calculating c_1 we will consider the boundary conditions of no-slip and heat-insulatedness on the surface of the vibrating plate (at $z = 0$)

$$z_1 = f = 0,$$
$$z_3 = \varphi = -ic,$$
$$z_6 = \theta' = 0.$$

Then, if we substitute the boundary conditions in (4.1), we obtain

$$c_1 f_r(0) + c_2 f_v(0) + c_3 f_t(0) = 0,$$
$$c_1\varphi_r(0) + c_2\varphi_v(0) + c_3\varphi_t(0) = -ic,$$
$$c_1\theta_r'(0) + c_2\theta_v'(0) + c_3\theta_t'(0) = 0. \tag{4.5}$$

For calculating c_1 we used the Kramer formula $c_1 = \Delta_1/\Delta$, where

$$\Delta_1 = \begin{vmatrix} 0 & f_v & f_t \\ -ic & \varphi_v & \varphi_t \\ 0 & \theta_v' & \theta_t' \end{vmatrix}, \quad \Delta = \begin{vmatrix} f_r & f_v & f_t \\ \varphi_r & \varphi_v & \varphi_t \\ \theta_r' & \theta_v' & \theta_t' \end{vmatrix}.$$

Here, we imply that the values of the functions are taken at $z = 0$. Then, it is necessary to substitute here the velocity and temperature functions in the form of expansion (4.3). Taking into account the fact that the values of g_0 were already found for the viscous and temperature cases (3.5) and (3.6), then there follows

$$c_1 = \frac{-ic}{\varphi_{inv}} + \frac{-ic}{\varphi_{inv}}\left(\frac{f_{inv}}{\varphi_{inv}}\frac{\varphi_v^1}{f_v^0}\right)\varepsilon + O(\varepsilon^2). \tag{4.6}$$

Using the system of equations for the inviscid approximation [18] and the expressions for φ_v^1 and f_v^0 from section 3.3 (f_{31} and f_{10}, respectively), we find

$$\frac{f_{inv}}{\varphi_{inv}} = i\frac{-\mathrm{M}^2(u-c)u'\varphi_{inv} + \varphi_{inv}'T}{\varphi_{inv}(T - \mathrm{M}^2(u-c)^2)}; \quad \frac{\varphi_v^1}{f_v^0} = \frac{-i}{g_0'} = -i\left(\sqrt{\frac{i}{\nu}(u-c)}\right)^{-1}.$$

We substitute these expressions in the formula (4.6) and use the boundary condition $\varphi_{inv}(0) = -ic$. Thus, the expression for c_1 takes the form:

$$c_1 = 1 + \left(\sqrt{\frac{i}{\nu}(u-c)}\right)^{-1} \frac{-\mathrm{M}^2(u-c)u'\varphi_{inv} + \varphi'_{inv}T}{\varphi_{inv}(T - \mathrm{M}^2(u-c)^2)}\varepsilon + O(\varepsilon^2).$$

As a result, using the expression for the pressure perturbation from the inviscid approximation (3.3), we find the final expression for the pressure perturbation

$$\pi(z) = \left[1 + \left(\sqrt{\frac{i}{\nu}(u-c)}\right)^{-1} \frac{-\mathrm{M}^2(u-c)u'\varphi_{inv} + \varphi'_{inv}T}{\varphi_{inv}(T - \mathrm{M}^2(u-c)^2)}\varepsilon\right]_{z=0} \times \left(-i\gamma \mathrm{M}^2 \left[\frac{(u-c)\varphi'_{inv} - u'\varphi_{inv}}{T - \mathrm{M}^2(u-c)^2}\right]\right) + O(\varepsilon^2). \tag{4.7}$$

In the case of the real value of c the question of choosing the branch of the root in (4.7) arises. Earlier, it was found (see (3.5)) that

$$g_0 = \int\limits_{z^*}^{z} \sqrt{\frac{i}{\nu}(u-c)}dz.$$

In this case the branch of the root is chosen such that the radiation condition at infinity must be satisfied: $\mathrm{Re}g_0(z) < 0$ (and, consequently, also $\mathrm{Re}g'_0(z) < 0$) as $z \to \infty$.

Initially, we will consider the case of the phase velocity lying on the interval $0 < c < \mathrm{M}$ and calculate the value of g'_0 at $z = 0$. At the large z the radicand takes the form iQ, where Q is a positive real value; consequently, at the large z it is necessary to take the branch with the argument $-3\pi/4$. As z decreases at the turning point z_c, at which $u(z) - c = 0$, the radicand is equal to zero and the root has the branch point. The WKB solutions are invalid in the neighborhood of the turning point. In order to avoid the singularity it is necessary to represent the phase velocity in the form $c = \mathrm{Re}(c) + s \cdot i$, where s is a small positive quantity (the point z_c must be passed around from below to obtain the physically correct solution, see section 3.2). In Figs. 3,a and 3,b we have reproduced, respectively, the values of the functions $u(z)$ and $u(z) - c$ in complex planes and the arrows show variations in the functions as z decreases from δ to 0. In Fig. 3,c we have reproduced the values of two branches of $\sqrt{i(u-c)/\nu}$. It is required to choose the branch which is located to the left of the imaginary axis (the arrows show the path as z decreases from δ to zero). Then, decreasing z to zero, we can find the argument of the branch at the required point $g'(0)$ (Fig. 3,c). When s tends to zero we can see that $g'(0) = \sqrt{c/\nu}e^{3\pi i/4}$.

The cases $c < 0$ and $c > \mathrm{M}$ can be considered similarly, except for the fact that in these cases there is no need to add a small quantity $s \cdot i$ (Fig. 3,d). Consequently, we obtain

$$g'(0) = \sqrt{\left|\frac{c}{\nu}\right|}e^{-i\frac{3\pi}{4}}, \quad c < 0; \quad g'(0) = \sqrt{\frac{c}{\nu}}e^{i\frac{3\pi}{4}}, \quad c > \mathrm{M}. \tag{4.8}$$

5. DISPERSION RELATION FOR THE INFINITE PLATE IN GAS

To be specific, we will consider the case $c > 0$ (the case $c < 0$ will be investigated below). Consequently, in accordance with the previous section, $g'(0) = \sqrt{c/\nu}e^{3\pi i/4}$. For the further analysis it is convenient to nondimensionalize the expression (4.7) in another way. The velocities are nondimensionalized by the sonic speed a^*_∞ in outer steady-state flow, the lengths by the plate thickness, and the pressure by the quantity $\rho^*_{\mathrm{pl}}a^{*2}_\infty$, where ρ^*_{pl} is the plate density. As before, the temperature is nondimensionalized by the temperature in outer flow. As a result, using simple algebra, we obtain the expression

$$\pi(z) = \left[1 + \varepsilon\left|\frac{c}{\nu}\right|^{-\frac{1}{2}} e^{-i\frac{3\pi}{4}}(\mathrm{M}\delta)^{\frac{1}{2}}\left(-\frac{u'(0)}{c} + \frac{T(0)}{\mu c^2}\pi_{inv}(0)\right)\right]\pi_{inv}(z) + O(\varepsilon^2), \tag{5.1}$$

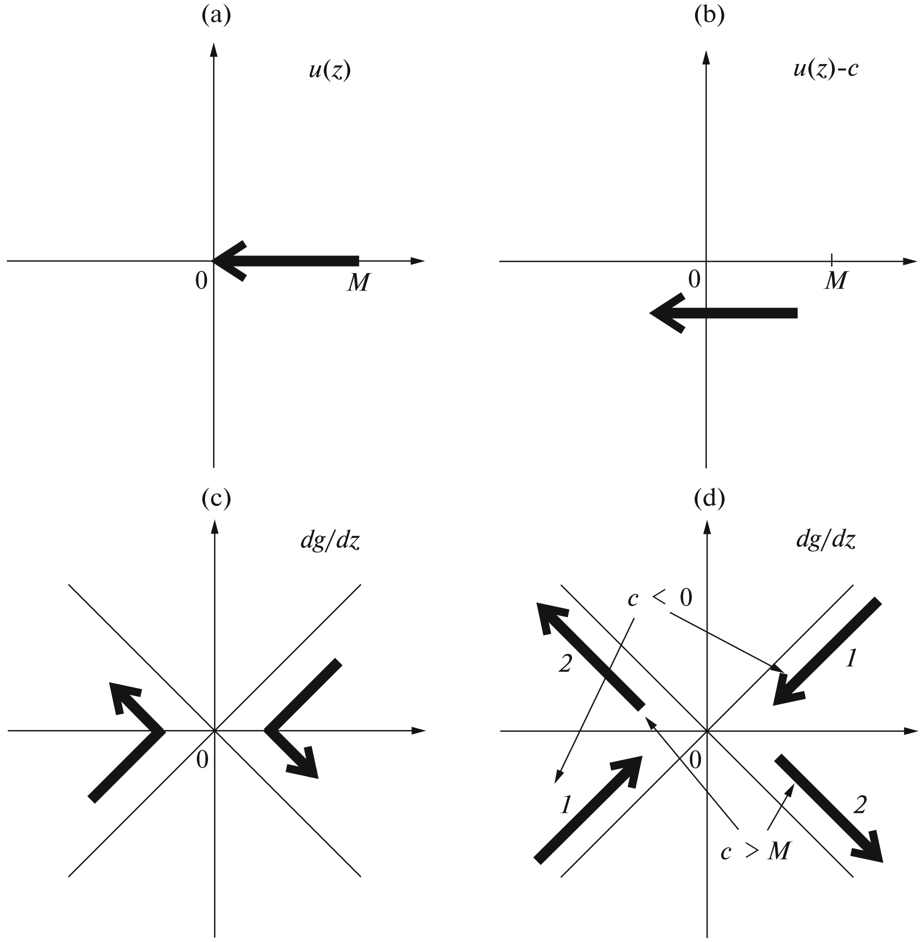

Fig. 3. The complex planes $u(z)$ (a), $u(z) - c$ (b), $\sqrt{\frac{i}{\nu}(u-c)}$ when $0 < c < \mathrm{M}$ (c); and $\sqrt{\frac{i}{\nu}(u-c)}$ when (curves *1* and *2*, respectively) $c < 0$ and $c > \mathrm{M}$ (d).

$$\pi_{inv}(z) = -\frac{i\mu}{\alpha}\left[\frac{(u-c)\varphi'_{inv} - u'\varphi_{inv}}{T - \mathrm{M}^2(u-c)^2}\right], \tag{5.2}$$

where the parameter μ is the ratio of the density of main flow to the density of the plate material. The Reynolds number entering into $\varepsilon = 1/\sqrt{\alpha \mathrm{R}}$ is taken in the form $\mathrm{R} = u^*_\infty \delta^*/\nu^*_\infty$ and ν is nondimensionalized by the density ν^*_∞ of flow outside the boundary layer.

In dimensionless variables the equation of motion of the plate [21] takes the form:

$$D\frac{\partial^4 w}{\partial x^4} - M_w^2\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial t^2} + p_d(x,t,z=0) = 0, \tag{5.3}$$

where D is the stiffness of the plate, the parameter M_w characterizes its tension, and p_d is the gas pressure perturbation.

Substituting the deflection of the plate in the form of traveling wave $w(x,t) = e^{i(\alpha x - \omega t)}$ and $p_d(x,t,z=0) = \pi(0)e^{i(\alpha x-\omega t)}$ in Eq. (5.3), we obtain the general form of the dispersion relation

$$F(\alpha,\omega) = D\alpha^4 + M_w^2\alpha^2 - \omega^2 + \pi(0) = 0. \tag{5.4}$$

Then, substituting the expression (5.1) for the pressure perturbation $\pi(0)$ in (5.4), we determine the explicit form of the dispersion relation:

$$F(\alpha,\omega) = D\alpha^4 + M_w^2\alpha^2 - \omega^2$$
$$+\left[1 + \varepsilon\left|\frac{c}{\nu}\right|^{-\frac{1}{2}} e^{-i\frac{3\pi}{4}}(\mathrm{M}\delta)^{\frac{1}{2}}\left(-\frac{u'(0)}{c} + \frac{T(0)}{c^2}\Pi_{inv}(0)\right)\right]\Pi_{inv}(0)\mu = 0,$$

where for the sake of convenience we introduced the notation

$$\Pi_{inv} = \frac{\pi_{inv}}{\mu} = -\frac{i}{\alpha}\left[\frac{(u-c)\varphi'_{inv} - u'\varphi_{inv}}{T - \mathrm{M}^2(u-c)^2}\right].$$

The solution $\omega(\alpha)$ of the dispersion relation can be found under the assumptions that $\mu \ll |\alpha|$ and $\mu \ll |\omega|$ valid for single-mode flutter [2]. After using the Taylor formula, we obtain

$$\omega(\alpha,\mu) = \omega(\alpha,0) + \mu\left[\frac{\partial\omega}{\partial\mu}\right]_{\mu=0} + o(\mu) = \omega(\alpha,0) - \mu\left[\frac{\partial F}{\partial\mu}\left(\frac{\partial F}{\partial\omega}\right)^{-1}\right]_{\mu=0} + o(\mu); \qquad (5.5)$$

hence, correct to small terms of the order of μ, the following equality must be satisfied

$$\omega(\alpha,\mu) = \omega(\alpha,0) + \frac{1}{2\omega(\alpha,0)}\left[1 + \varepsilon\left|\frac{c}{\nu}\right|^{-\frac{1}{2}} e^{-i\frac{3\pi}{4}}(\mathrm{M}\delta)^{\frac{1}{2}}\left(-\frac{u'(0)}{c} + \frac{T(0)}{c^2}\Pi_{inv}(0)\right)\right]\Pi_{inv}(0)\mu, \quad (5.6)$$

the expression in the square brackets being calculated at $\mu = 0$.

6. INFLUENCE OF VISCOUS PERTURBATIONS ON STABILITY OF THE PLATE FOR THE SMALL BOUNDARY LAYER THICKNESS

First of all, of interest is the fact what are the conditions under which the viscous term in (5.6) has the destabilizing or stabilizing effect. This depends on sign of the imaginary component of the viscous part of the solution (5.6), i.e., on sign of the imaginary part

$$\left[\left|\frac{c}{\nu}\right|^{-\frac{1}{2}} e^{-i\frac{3\pi}{4}}(\mathrm{M}\delta)^{\frac{1}{2}}\left(-\frac{u'(0)}{c} + \frac{T(0)}{c^2}\Pi_{inv}(0)\right)\right]\Pi_{inv}(0)\mu. \qquad (6.1)$$

In the case of the small boundary layer thickness, this investigation can be carried out in the general form. In what follows, we will consider the result of mapping (6.1) at $\delta = 0$ since for small $\delta > 0$ the values of (6.1) will be located in a small neighborhood of its values at $\delta = 0$.

When $\delta = 0$ the pressure perturbation is $\Pi_{inv}(0) = -A^{-1}$ [13], where

$$A = \frac{\sqrt{\alpha^2 - (\mathrm{M}\alpha - \omega)^2}}{(\mathrm{M}\alpha - \omega)^2} = \frac{\sqrt{1 - (\mathrm{M} - c)^2}}{\alpha(\mathrm{M} - c)^2}.$$

The value of (6.1) depends on the phase velocity c. Initially, we will assume that the perturbation grows in homogeneous flow in the absence of the boundary layer, i.e., when $0 < c < \mathrm{M} - 1$, $\mathrm{Im}\,A = a > 0$, and $\mathrm{Re}A = 0$. Consequently, $\Pi_{inv}(0) = a^{-1}i$. From (6.1) we can see than the effect of the viscous term is determined by sign of the imaginary part of the expression

$$V = e^{-i\frac{3\pi}{4}}\Pi_{inv}(0)\left(K\Pi_{inv}(0) - 1\right), \quad K = \frac{T(0)}{u'(0)c} > 0. \qquad (6.2)$$

The viscous term has the destabilizing or stabilizing effect when $\mathrm{Im}\,V > 0$ and $\mathrm{Im}\,V < 0$, respectively.

In Fig. 4,a we have reproduced the result of the mapping (6.2) for all possible values of a. Hence we can see that $\mathrm{Im}\,V > 0$, i.e., for the small boundary layer thicknesses the viscous term always leads to increase in the perturbation growth rates.

We will now consider the waves of the following type: the wave is neutral in homogeneous flow in the absence of the boundary layer, i.e., $\mathrm{M} - 1 < c < \mathrm{M} + 1$. In this case $A \in \mathbb{R}$ so that $A = a > 0$. The results of the mapping (6.2) for various a (Fig. 4,b) show that the viscous term has the stabilizing effect.

We will now assume that the perturbation is damped in homogeneous flow in the absence of the boundary layer, i.e., $c < 0$ or $c > \mathrm{M} + 1$. Initially, we will consider the case $c > \mathrm{M} + 1$. Then A is purely imaginary so that $\mathrm{Im}\,A = a < 0$. In Fig. 4,c we have reproduced the result of application of (6.2). The result depends on the multiplier K: for the small boundary layer thickness, regardless of the boundary layer profile, the term of the order of ε has the stabilizing and destabilizing effect when $K < |a|$ and $K > |a|$, respectively.

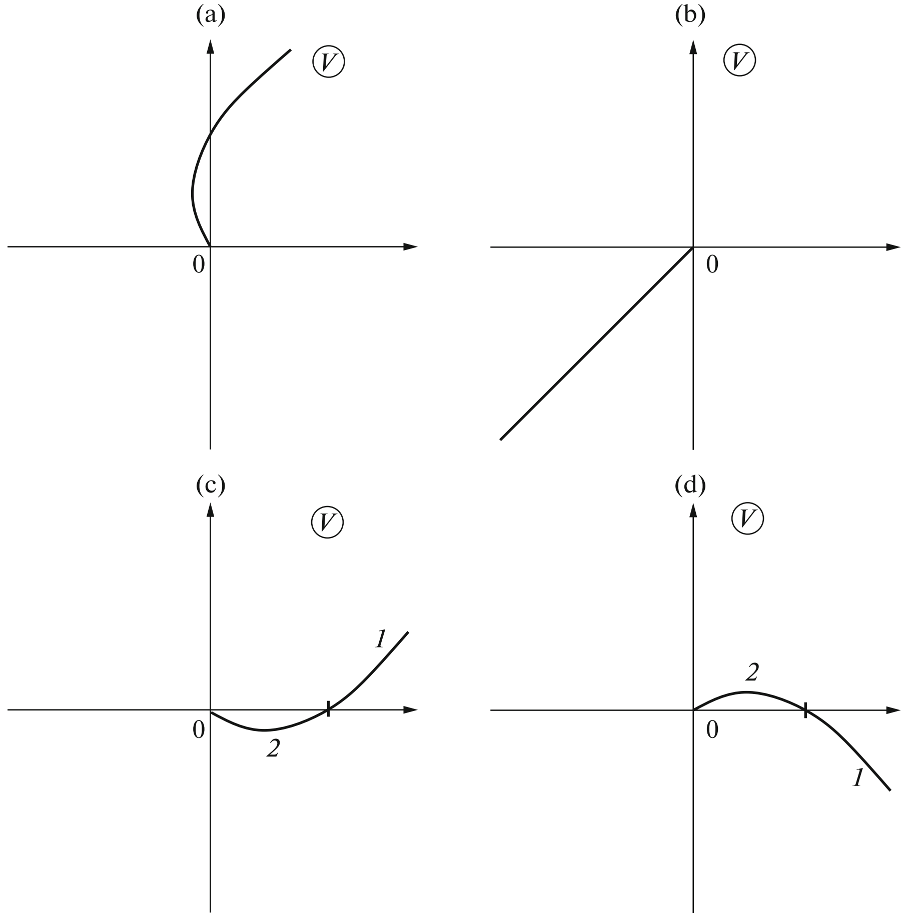

Fig. 4. Result of the mapping V (6.2) in the cases of $0 < c < \mathrm{M} - 1$ (a), $\mathrm{M} - 1 < c < \mathrm{M} + 1$ (b), $\mathrm{M} + 1 < c < \infty$ (c) for $|K| > |a|$ (curve *1*) and $|K| < |a|$ (curve *2*) (c); result of the mapping V (6.3) in the case of $c < 0$ for $|K| > |a|$ (curve *1*) and $|K| < |a|$ (curve *2*) (d).

Let now $c < 0$. As before, in this case A is purely imaginary and $\operatorname{Im} A = a < 0$. Here, the expression for the viscous coefficient differs from all previous cases since, in accordance with the root choice rule (section 4), it is necessary to use the multiplier $e^{3\pi i/4}$ instead of $e^{-3\pi i/4}$. Thus, the viscous term takes the form:

$$V = e^{i\frac{3\pi}{4}} \Pi_{inv}(0) \left(K \Pi_{inv}(0) - 1\right), \quad K < 0. \tag{6.3}$$

As a result of applying the mapping (6.3) to various a, we obtain the pattern shown in Fig. 4,d. For the small boundary layer thickness (for any boundary layer profile) the term of the order of ε has the stabilizing effect if $|K| > |a|$ and the destabilizing effect if $|K| < |a|$.

7. BOUNDARY LAYER OF AN ARBITRARY THICKNESS

7.1. The Case of the Long Waves

In the general case of the finite boundary layer thickness the Rayleigh equation cannot be solved analytically; therefore, calculation of φ_{inv} in (4.7) requires to use numerical methods. In the present paper the qualitative effect of viscosity is investigated; therefore, in what follows we will restrict our attention to the longwave approximation when the Rayleigh equation can be solved analytically [13, 15].

Neglecting the term α^2 in the Rayleigh equation, we obtain

$$\frac{d}{dz}\left(\frac{(u-c)\varphi'_{inv} - u'\varphi_{inv}}{T - (u-c)^2}\right) = 0.$$

The general form of the solution of this equation takes the form:

$$\varphi_{inv}(z) = \left(d_1\left(\int_0^z \frac{T}{(u-c)^2}dz - z\right) + d_2\right)(u-c),$$

where d_1 and d_2 are constants and, respectively, the pressure perturbation is equal to

$$\pi_{inv}(z) = -\frac{i\mu d_1}{\alpha}.$$

The constants d_1 and d_2 can be calculated using the boundary conditions (3.4).

To be specific, let the wave be traveling forward, i.e., $c > 0$. Using (5.1), the expression for $\pi(0)$ can be found correct to $O(\varepsilon^2)$

$$\begin{aligned}\pi(0) = -\mu &\left(\left(\frac{(\mathrm{M}\alpha-\omega)^2}{\sqrt{\alpha^2-(M\alpha-\omega)^2}}\right)^{-1} + \left(\int\limits_0^\delta \frac{Tdz}{(u-c)^2} - \delta\right)\right)^{-1} \\ &\times \left(1 - \varepsilon\left|\frac{c}{\nu}\right|^{-\frac{1}{2}} e^{-i\frac{3\pi}{4}}(\mathrm{M}\delta)^{\frac{1}{2}}\left[\frac{u'}{c} + \frac{T}{c^2}\left(\left(\frac{(\mathrm{M}\alpha-\omega)^2}{\sqrt{\alpha^2-(\mathrm{M}\alpha-\omega)^2}}\right)^{-1}\right.\right.\right. \\ &\left.\left.\left. + \left(\int\limits_0^\delta \frac{Tdz}{(u-c)^2} - \delta\right)\right)^{-1}\right]\right).\end{aligned} \tag{7.1}$$

Then for the long waves the solution of the dispersion relation (5.5) takes the form:

$$\begin{aligned}\omega(\alpha,\mu) = \omega(\alpha,0) - \frac{\mu}{2\omega(\alpha,0)} &\left(\left(\frac{(\mathrm{M}\alpha-\omega)^2}{\sqrt{\alpha^2-(\mathrm{M}\alpha-\omega)^2}}\right)^{-1} + \left(\int\limits_0^\delta \frac{Tdz}{(u-c)^2} - \delta\right)\right)^{-1} \\ &\times \left\{1 - \varepsilon\left|\frac{c}{\nu}\right|^{-\frac{1}{2}} e^{-i\frac{3\pi}{4}}(\mathrm{M}\delta)^{\frac{1}{2}}\left[\frac{u'}{c} + \frac{T(0)}{c^2}\left(\left(\frac{(\mathrm{M}\alpha-\omega)^2}{\sqrt{\alpha^2-(\mathrm{M}\alpha-\omega)^2}}\right)^{-1}\right.\right.\right. \\ &\left.\left.\left. + \left(\int\limits_0^\delta \frac{Tdz}{(u-c)^2} - \delta\right)\right)^{-1}\right]\right\},\end{aligned} \tag{7.2}$$

in all the brackets the expressions being calculated at $\mu = 0$.

7.2. Investigation of the Growing Waves

In what follows, similarly to Section 6, we will consider various ranges of the phase velocities of the waves. Initially, we take the case $0 < c < \mathrm{M} - 1$, $\mathrm{Im}\, A = a > 0$, and $\mathrm{Re} A = 0$.

The effect of the term of the order of ε on the perturbation growth rate will be investigated using the formula (7.2)

$$\omega(\alpha,\mu) = \omega(\alpha,0) - \frac{\mu}{2\omega(\alpha,0)}(A+B)^{-1}\left[1 - \varepsilon e^{-i\frac{3\pi}{4}}K_1\left[K_2 + K_3(A+B)^{-1}\right]\right]_{\mu=0},$$

$$B = \delta\left(\int\limits_0^1 \frac{T(\eta)d\eta}{(u(\eta)-c)^2} - 1\right),$$

$$K_1 = \left|\frac{c}{\nu}\right|^{-\frac{1}{2}}(\mathrm{M}\delta)^{\frac{1}{2}} > 0; \quad K_2 = \frac{u'(0)}{c} > 0; \quad K_3 = \frac{T(0)}{c^2} > 0.$$

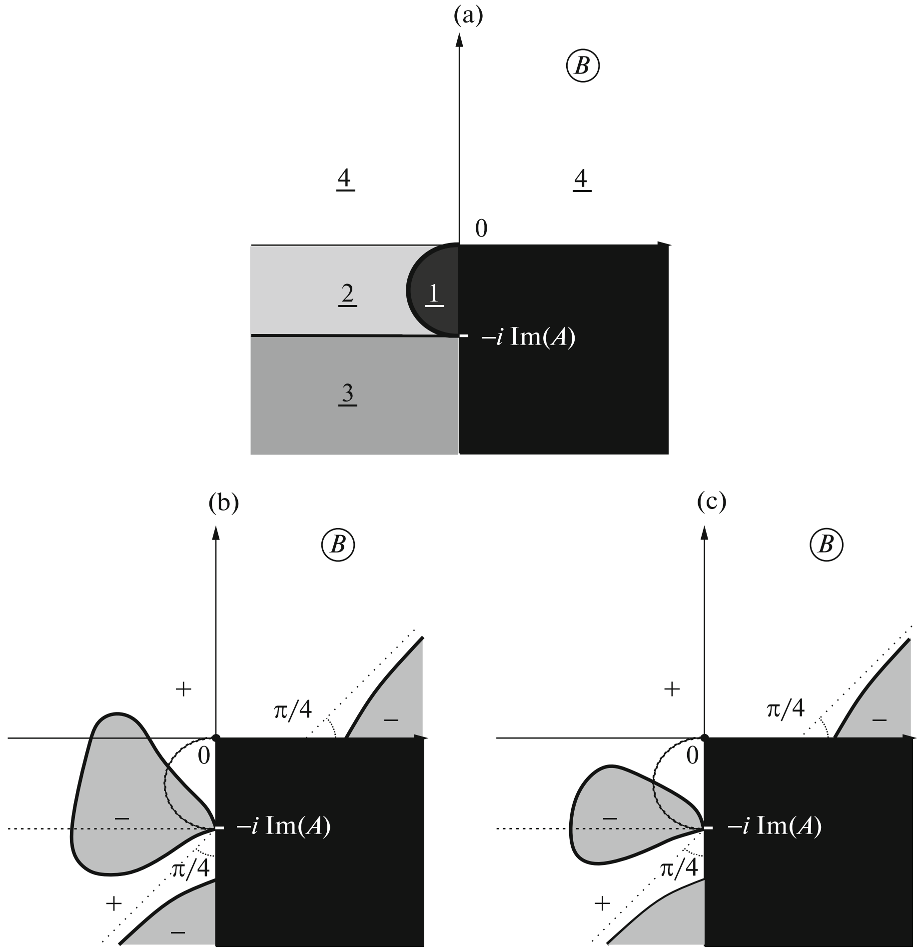

Fig. 5. Complex plane B divided into domains (a); the values of B which fall into the lower (upper) half-plane in the mapping $V(B)$ (7.3) are denoted by grey (white) color in the cases of $2a \ll K$ (b) and $2a \gg K$ (c).

The effect of viscosity can be determined from the expression

$$V = (A+B)^{-1} e^{-i\frac{3\pi}{4}} \left[1 + K(A+B)^{-1}\right]_{\mu=0}; \quad K = \frac{K_3}{K_2} = \frac{T(0)}{u'(0)c} > 0. \tag{7.3}$$

Viscosity has the destabilizing effect when $\operatorname{Im} V > 0$ and the stabilizing effect when $\operatorname{Im} V < 0$. Introducing the notation $G = (A+B)^{-1}$, we can write the expression (7.3) in the form:

$$V = Ge^{-i\frac{3\pi}{4}} \left[1 + KG\right]_{\mu=0}. \tag{7.4}$$

From these formulas we can see that the quantity B depends linearly on the boundary layer thickness δ and $B \to 0$ as $\delta \to 0$.

In investigating the term of the order of ε, of interest are the ranges of value of the expression (7.3) for various B. For this purpose the complex plane B was divided into several domains (Fig. 5,a; the domain $\operatorname{Re}(B) > 0, \operatorname{Im}(B) < 0$ does not satisfy the stability criterion of supersonic flows [13, 15] and was not investigated). The values of the expression (7.3) were constructed and analyzed individually for each of the domains.

As an example, we will consider domain 1 (Fig. 5,a; the other domains can be similarly investigated) in which the following constraints must be additionally satisfied [13, 15]:

1. In the presence of the boundary layer, as $\mathrm{R} \to \infty$, the perturbations grow more rapidly than in homogeneous flow, i.e., $\operatorname{Im}(A+B)^{-1} < \operatorname{Im} A^{-1}$,

2. Re$B < 0$; this is the necessary condition of stability of the boundary layer on the absolutely rigid plate.

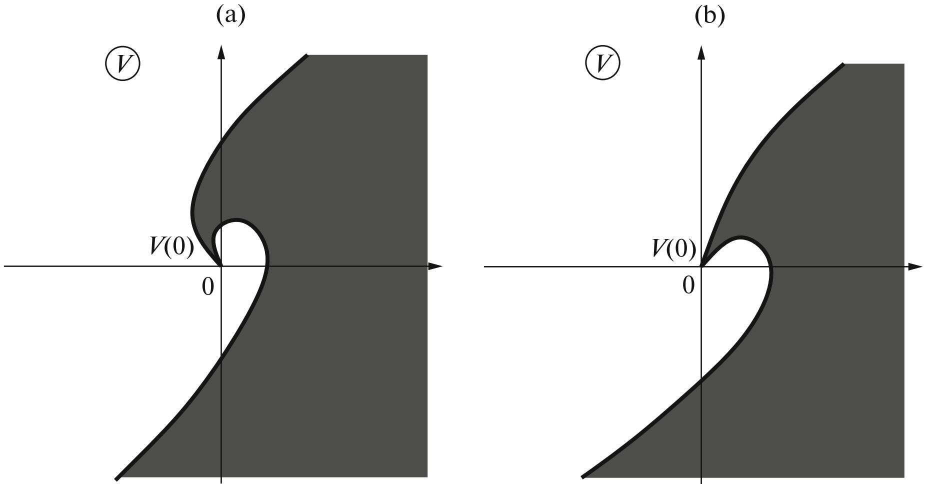

Fig. 6. Domain of the value of $V(B)$ (7.3) in the mapping of domain "1" of the complex plane B (Fig. 5) in the cases of $a > K$ (a) and $a < K$ (b).

As a result of application (7.3) to this domain of B, we obtain the domain of values of the term of the order of ε shown in Figs. 6,a and 6,b. Since $\operatorname{Im} V(0) > 0$ in the neighborhood of $B = 0$, hence we can see that for small δ the effect of viscosity always implies increase in the perturbation growth rate. This corresponds to the results of Section 6. When δ increases there exist domains of the values of the multiplier of the order of ε at which viscosity has both the stabilizing effect (the perturbation growth rate decreases) and the destabilizing effect (the perturbation growth rate increases).

Of interest is the fact which of the values of B fall into the upper half-plane $\operatorname{Im} V(B) > 0$ in the mapping (7.3). In Figs. 5,b and 5,c we have reproduced the result of application of (7.3) to various B. In these figures the values of B which fall into the upper and lower half-planes in the mapping $V(B)$ (7.3) are denoted by "+" and "−", respectively. The result of the mapping (7.3) depends on K: in the case of $2a \ll K$ we obtain the pattern shown in Fig. 5,b and in the case of $2a \gg K$ that shown in Fig. 5,c. Thus, the multiplier K determines whether the part of the domain of B, in which $\operatorname{Re} B < 0$ and $\operatorname{Im} B > 0$, falls into the lower half-plane in the mapping (7.3) or this entire domain falls into the upper half-plane.

In [13, 15] it was shown that

$$\operatorname{Im} B = -\pi\delta \frac{T^2}{u'^3}\left(\frac{u'}{T}\right)',$$

the functions on the right-hand side being taken at the critical point. Thus, in the case of the generalized convex profiles we always have $\operatorname{Im} B > 0$. In the case of the profiles with the generalized inflection point the sign of $\operatorname{Im} B$ depends on the phase velocity of perturbations c.

As examples, we will now calculate the effect of viscous perturbations for particular boundary layer profiles: the generalized convex profile and that with the generalized inflection point which correspond to [16]. In both examples we took the wavenumber $k = 0.005$.

As the first example, we take the generalized convex profile

$$u(z) = \mathrm{M} \sin\left(\frac{\pi}{2}\frac{z}{\delta}\right) \tag{7.5}$$

with the parameters $\mathrm{M} = 1.6$ and

$$D = 23.9, \quad M_w = 0, \quad \gamma = 1.4. \tag{7.6}$$

These parameters correspond to a steel plate in the air stream at the altitude of 3 km or an aluminum plate at the altitude of 11 km. To be specific, we will assume that the Prandtl number $\mathrm{Pr} = 1$ and the plate is heat-insulated; consequently, the temperature profile $T(u)$ can be given by the same expression as that in adiabatic flow [22]

$$T(u) = 1 + \frac{\gamma - 1}{2}(\mathrm{M}^2 - u^2). \tag{7.7}$$

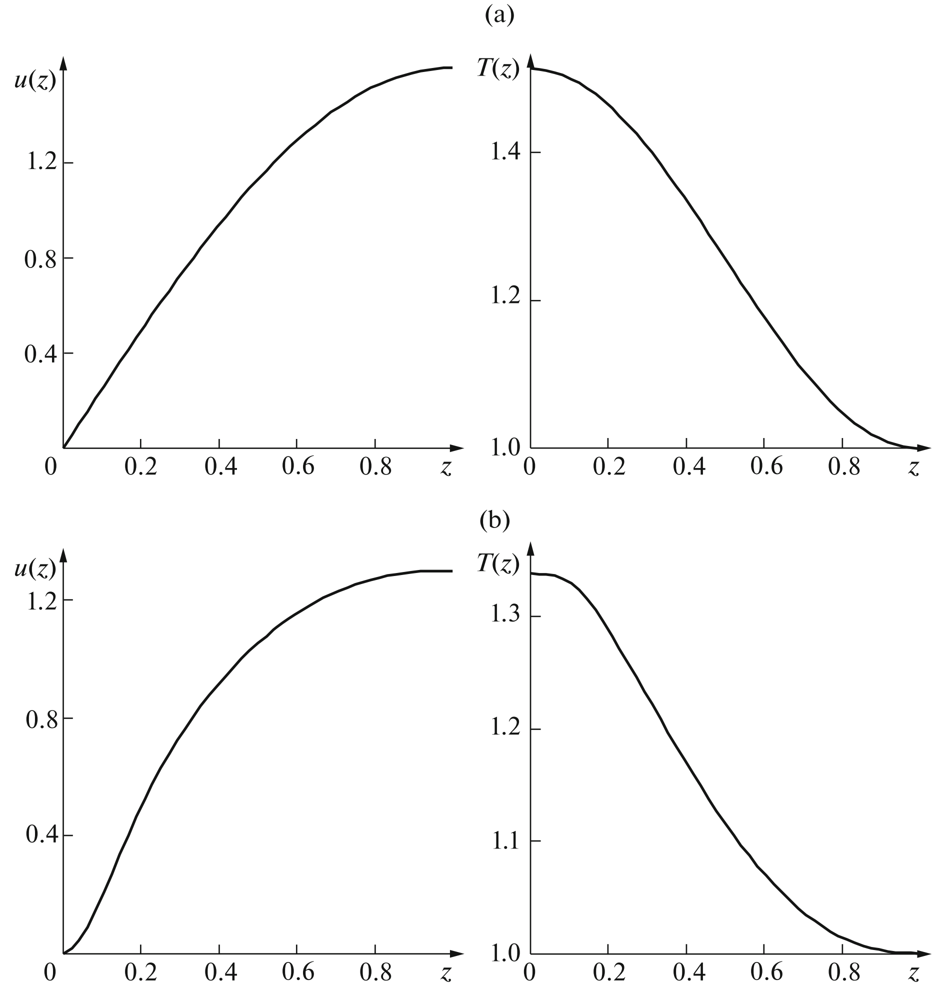

Fig. 7. Effect of viscous perturbations: the generalized convex profile of the boundary layer (a); the profile with the generalized inflection point (b).

In Fig. 7,a we have reproduced the velocity and temperature profiles (7.5) and (7.7). This boundary layer is stable on the absolutely rigid plate in the inviscid approximation [16].

As a result, we obtain $B = (-25.815 + 0.057i)\delta$, $a = 98.094$, and $K = 24.61$. The value of $\operatorname{Im} V$ of the expression (7.3) is positive for any boundary layer thickness not equal to zero, so that it increases on the interval of variation in δ from 0 to $\delta_1 = 0.57$ and for $\delta > \delta_1$ it decreases and tends to zero as $\delta \to \infty$ (Fig. 8,a). The calculation data correspond to the pattern in Fig. 5,c in the case of $2a \gg K$.

As the second example, we will consider the profile with the generalized inflection point

$$u(z) = \mathrm{M}\left(1 - \left(1 - \frac{z}{\delta}\right)^{2.4}\right)\cos\left(0.7\left(1 - \frac{z}{\delta}\right)^{7}\right)^{7} \tag{7.8}$$

with $\mathrm{M} = 1.3$, the parameters (7.6), and the temperature profile (7.7) shown in Fig. 7,b. The velocity profile (7.8) represents a function with a single generalized inflection point located in the supersonic part of the layer. In the inviscid approximation on the absolutely rigid surface the stability of this boundary layer was shown in [16].

As a result, we obtain $B = (-5.712 - 21.249i)\delta$, $a = 48.033$, and $K = 57.29$. The value of $\operatorname{Im} V$ takes the following form (Fig. 8,b): $\operatorname{Im} V > 0$ when $0 < \delta < \delta_{21} = 1.56$, $\operatorname{Im} V < 0$ when $\delta_{21} < \delta < \delta_{22} = 2.49$, $\operatorname{Im} V > 0$ when $\delta_{22} < \delta < \delta_{23} = 8.29$, and $\operatorname{Im} V < 0$ when $\delta > \delta_{23}$. The calculation data correspond to the pattern in Fig. 5,b.

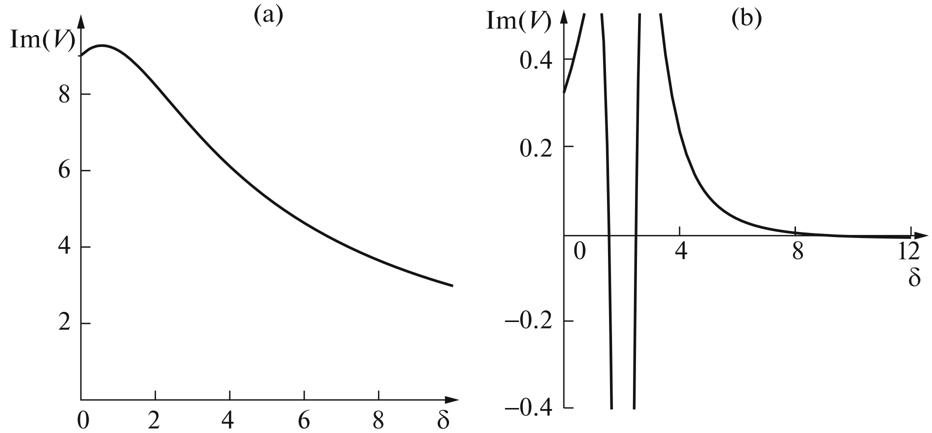

Fig. 8. Imaginary part of the viscous term of the order of ε as a function of the boundary layer thickness $\operatorname{Im} V(\delta)$ in the case of the generalized convex profile (7.5), (7.6), and (7.7) (a) and in the case of the profile with the generalized inflection point (7.8), (7.6), and (7.7) (b).

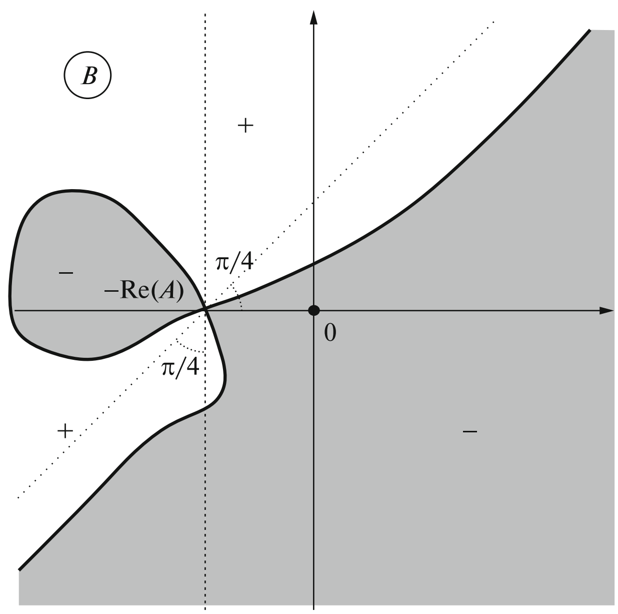

Fig. 9. Values of B which fall into the lower (upper) half-plane in the mapping $V(B)$ (7.3) are denoted by grey (white) color.

7.3. Investigations of the Neutral and Damped Waves

The waves neutral in homogeneous flow correspond to the phase velocities $\mathrm{M} - 1 < c < \mathrm{M} + 1$. Initially, we will consider the waves with the phase velocities on the interval $\mathrm{M} - 1 < c < \mathrm{M}$. In this case, as in section 7.2, B is the complex quantity which depends linearly on δ, but $A \in \mathbb{R}$ such that $A = a > 0$.

Similarly to section 7.2, we have constructed the domains of the values of the expression $V(B)$ (7.3) for various B in order to investigate the effect of the term of the order of ε. The result is shown in Fig. 9 in which the domains of B for which $\operatorname{Im} V(B)$ is positive or negative are denoted by "+" and "−", respectively.

Hence we can see that for small δ taking viscosity into account implies a decrease in the perturbation growth rate. This corresponds to section 6. As δ increases, there are domains of B in which viscosity has both stabilizing and destabilizing effect.

Let now $\mathrm{M} < c < \mathrm{M} + 1$. In this case A is real, $A = a > 0$, and B is also real since there is no critical point z_c, where $u(z_c) = c$. Consequently, the domain of values of B is the whole real axis. As a result

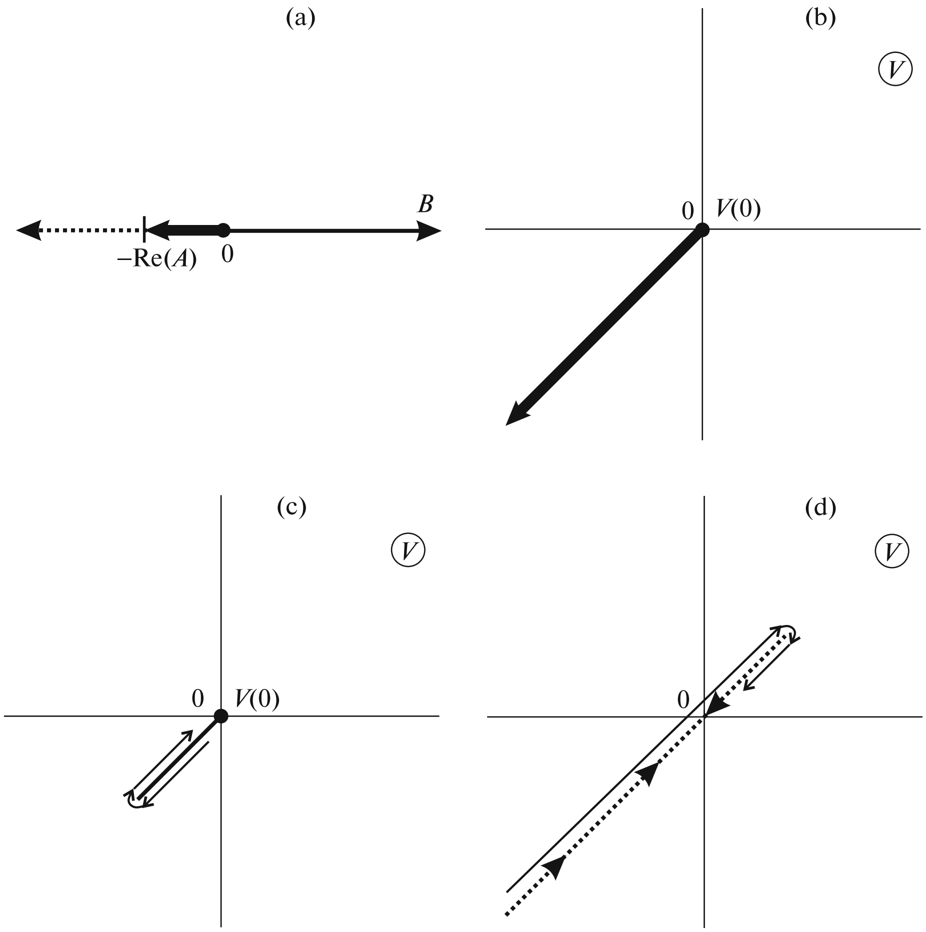

Fig. 10. Real axis of B in the case of $\mathrm{M} < c < \mathrm{M} + 1$ divided into parts (a); results of the mapping $V(B)$ for different parts of the real axis of B (b)–(d).

of application of the mapping (7.3) to all the values of B which are possible in this case (Fig. 10,a), we obtain the patterns shown in Figs. 10, b–d.

The results show that the viscous term has the stabilizing effect for small boundary layer thicknesses, whereas for large thicknesses the effect can be opposite.

Finally, we will consider the waves which damp in the absence of the boundary layer in homogeneous flow, i.e., when $c < 0$ or $c > \mathrm{M} + 1$. Initially, let $c > \mathrm{M} + 1$. Then B is real and A is purely imaginary, such that $\operatorname{Im} A = a < 0$.

In Figs. 11, a–c we have reproduced the results of application of (7.3) to various B. We can see that the results depend of the multiplier K: for the small boundary layer thickness the term of the order of ε has the stabilizing effect regardless of the boundary layer profile when $K < |a|$ and the destabilizing effect when $K > |a|$. As the boundary layer thickness increases, taking viscosity into account can lead to both stabilization and destabilization of the system depending on the value of B.

Let now $c < 0$. As before, in this case B is real and A is purely imaginary, $\operatorname{Im} A = a < 0$. Here, the expression for the viscous term differs from all previous cases since, in accordance with section 4, in (7.3) it is necessary to use the multiplier $e^{3\pi i/4}$ instead of $e^{-3\pi i/4}$. Thus, the viscous term takes the form:

$$V = (A + B)^{-1} e^{i\frac{3\pi}{4}} \left[1 + K (A + B)^{-1}\right]_{\mu=0}, \qquad (7.9)$$

the coefficient K becoming negative.

As a result of application of the mapping (7.9) to the real axis of B, we obtain the domains shown in Figs. 12, a–c. For small δ (for any boundary layer profiles) the term of the order of ε has the stabilizing

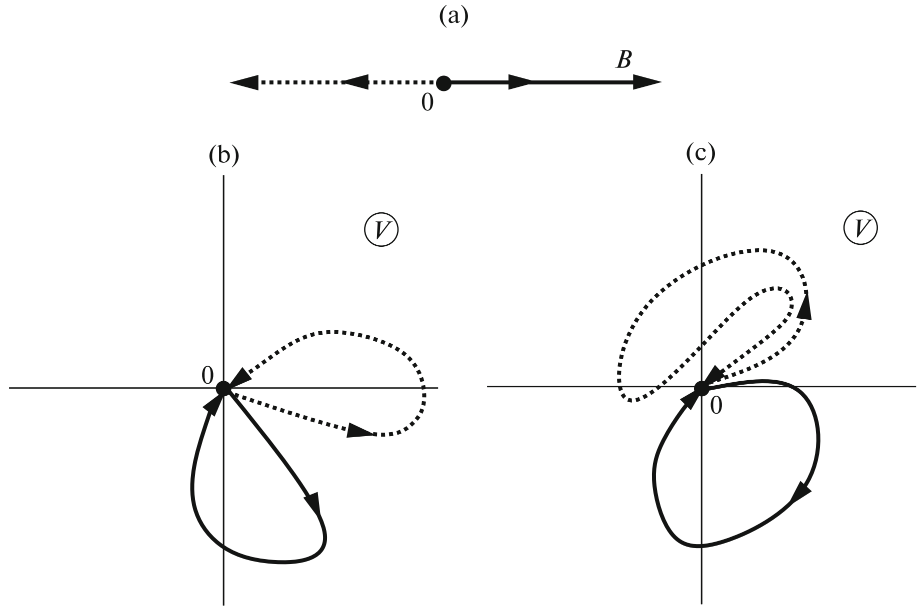

Fig. 11. Real axis of B in the case of $\mathrm{M}+1 < c < \infty$ divided into parts (a); domains of the values of $V(B)$ (7.3) in the cases of $K < |a|$ (b) and $K > |a|$ (c).

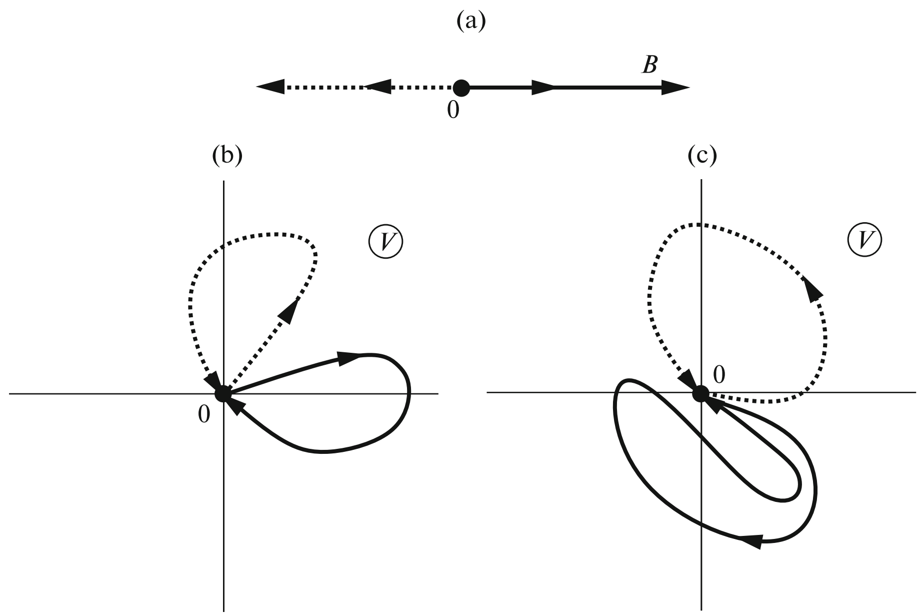

Fig. 12. Real axis of B in the case of $c < 0$ divided into parts (a); domains of the values of $V(B)$ (7.9) in the cases of $K < |a|$ (b) and $K > |a|$ (c).

effect when $|K| > |a|$ and the destabilizing effect when $|K| < |a|$. For large δ there exist such B for which the viscous term has both the stabilizing and destabilizing effect.

Summary. The effect of viscous and temperature perturbations of the boundary layer formed on the plate surface located in the gas flow on the behavior of traveling waves is investigated at large but finite Reynolds numbers.

The dispersion relation which describes the perturbation growth in the first approximation in $(\sqrt{\alpha \mathrm{R}})^{-1}$ is derived. The effect of viscosity is investigated in the general form for the small boundary layer thickness. It is shown that in this case the effect of the finite Reynolds number can be both destabilizing and stabilizing depending on the phase velocity of propagation of the perturbations.

The case of long waves is investigated analytically for the possible phase velocities and an arbitrary boundary layer thickness. Examples of the generalized convex boundary profile and the profile with the inflection point are considered for the long growing waves in homogeneous flow and the boundary layer thickness ranges in which the viscous term has the stabilizing and destabilizing effects are calculated.

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