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Technical Notes

Effect of Nonequilibrium Reacting Flow on Flutter at Hypersonic Flight Speed

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Nomenclature

a_∞	=	freestream speed of sound, m/s
h	=	panel thickness, m
K	=	piston theory coefficient; $\gamma p M^2/\sqrt{M^2-1}$, Pa
L, L_x	=	panel length, m
L_y	=	panel width, m
M	=	local Mach number
M_∞	=	freestream Mach number
p	=	local air pressure, Pa
p_∞	=	freestream air pressure, Pa
T	=	air temperature, K
w	=	deflection of the surface, m
γ	=	specific heat rate

I. Introduction

FLIGHT vehicles moving at subsonic and moderate supersonic speeds can experience flutter of various structures, such as wings, control surfaces, and skin panels. For a theoretical analysis of the flutter boundary, the stability of coupled linearized structural and aerodynamic models is studied. If the structure is unstable, the danger of flutter can be estimated by limit-cycle analysis, which requires a nonlinear structural model, whereas aerodynamic nonlinearity in most cases can be neglected [1,2]. However, at high supersonic and hypersonic speeds, the aerodynamic part of the problem becomes more complicated. First, aerodynamic nonlinearity becomes significant and affects the limit-cycle amplitude (but not the flutter boundary). Second, at hypersonic flow over slender bodies, viscous–inviscid interaction should be included into the aerodynamic model because the boundary-layer effect cannot be neglected. Finally, at hypersonic speeds, the air after the shock, consisting at normal conditions of O_2 and N_2 molecules (the presence of other components is minor and is neglected in this study), experiences dissociation and becomes a mixture of various species, namely, O, N, NO, O_2, and N_2, whose composition, being chemically nonequilibrium in general, spatially evolves.

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The first effect of hypersonic flow, aerodynamic nonlinearity, has been studied in literature by many authors [3–8]; this effect can yield a larger limit-cycle amplitude and a change from supercritical to subcritical Hopf bifurcation. However, the second and third effects, viscous–inviscid interaction and the nonequilibrium state of the air, to our knowledge, have not been studied previously in the context of flutter (except for [9], where real-gas effects were considered but turned out to be minor for the flow regime analyzed), and they can affect not only limit-cycle amplitude but also flutter boundary. In this study, we investigate the impact of the nonequilibrium reacting airflow on flutter boundary, ignoring the viscosity effect, which will be treated in a separate study.

II. Effect of Reacting Flow on Unsteady Aerodynamics

A. General Considerations

It is known that, at large supersonic speeds ($M \to \infty$), the plane section law is valid, which implies that, in the coordinate system connected to the gas (in which the gas is at rest), during the motion of a slender body, the flow particles move normally to the direction of the body motion, whereas the velocity component codirected with the body is negligible (Fig. 1). This means that the body acts in each plane occupied by the gas as a two-dimensional piston. Taking a closed-form solution for the piston, a pressure perturbation expressed through the body velocity is obtained:

$$p = p_\infty \left(1 + \frac{\gamma - 1}{2}\frac{v}{a_\infty}\right)^{2\gamma/(\gamma-1)} \tag{1}$$

Expressing the body velocity v through the deflection w in the coordinate system connected to the body, and linearizing, the well-known piston theory [3,10,11] for pressure perturbation is obtained:

$$\Delta p = p - p_\infty = \frac{\gamma p_\infty}{a_\infty}\left(U\frac{\partial w}{\partial x} + \frac{\partial w}{\partial t}\right) \tag{2}$$

At high speeds, the second term, $\partial w/\partial t$, which expresses aerodynamic damping, can be neglected compared to the first term, aerodynamic stiffness, which yields the quasi-static expression:

$$\Delta p = \gamma p_\infty M_\infty \frac{\partial w}{\partial x} \tag{3}$$

This formula shows a good correlation with the exact pressure calculations for Mach numbers 3 to 5 and larger but can be improved to include Mach number range $2 < M < 3$ [12] by the following modification:

$$\Delta p = \frac{\gamma p_\infty M_\infty^2}{\sqrt{M_\infty^2 - 1}}\frac{\partial w}{\partial x} \tag{4}$$

In this form, it coincides with Ackeret's formula [3] for pressure acting on a slender body in supersonic flow. Because Ackeret's formula is the exact solution of linearized flow equations, this is the "best" quasi-static expression and is taken in this study as the basis for analyzing the reacting flow effect.

It can be traced that, in the case of reacting mixture instead of a perfect gas, the plane section law stays valid because its derivation [11] uses only momentum equations but not thermodynamics. We will assume that, at each spatial location, the mixture can be considered as being in local equilibrium, and the expression in Eq. (4) stays valid, with γ, p_∞, and M_∞ substituted by its local values (Fig. 1). Although the use of local steady pressure and Mach number values is obvious and common in aeroelasticity, the change in the

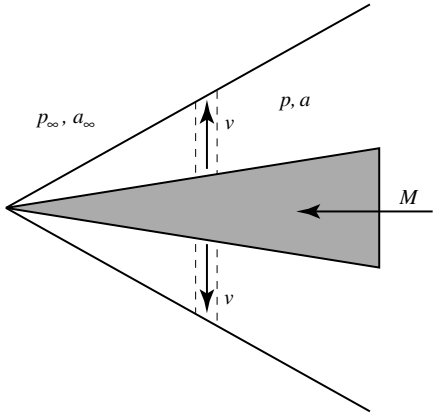

Fig. 1 Motion of the gas perpendicular to flight direction.

specific heat ratio is a purely hypersonic flow effect. Let us consider this in more detail.

When the flow particle crosses the bow shock, its temperature increases, and the mixture primarily consisting of molecular oxygen and nitrogen, O_2 and N_2, becomes chemically nonequilbrium because the equilibrium air state corresponding to the temperature behind the shock also consists of atomic oxygen O and nitrogen N as well as nitric oxide NO (other species and ionization effects are neglected in this study for simplicity). Hence, oxygen and nitrogen molecules start to dissociate, yielding the appearance of a significant amount of other species, which results in flow with a system of chemical reactions that tends downstream to the equilibrium state corresponding to the current temperature. On the other hand, during the motion of the particle, temperature also changes downstream so that the composition of the mixture at each point is the solution of the coupled system consisting of equations of motion and equations of chemical reactions.

We will assume that, for the calculation of pressure perturbation for flutter analysis purposes, the flow can be considered as locally equilibrium at each point. Then, the effect of the hypersonic flow consists of two points. First, in the quasi-static piston theory,

$$\Delta p = K(x,y)\frac{\partial w}{\partial x}, \qquad K(x,y) = \frac{\gamma p M^2}{\sqrt{M^2-1}} \tag{5}$$

the coefficient K is a function of the spatial location. For the case of a sufficiently short skin panel in nonreacting flow (so that the flow over the panel can be considered uniform), K is constant along the panel, which yields a classical coupled-mode flutter mechanism [3,12,13]. However, if the flow is essentially nonequilibrium, chemical reactions yield a significant change in K primarily because of the change in the mixture composition along the panel.

Second, the flow composition is changed because of the deflection of the surface. This change yields the change of the flow pressure p_∞ of the order of the surface deflection, that is, it should be taken into account in the linearized approximation for pressure perturbation. However, in the example considered later, we will show that this component of pressure perturbation is much smaller than that because of regular piston theory [Eq. (5)], which is why it can be neglected in most circumstances.

B. Conditions at Which the Reacting Flow Effect Is the Most Pronounced

We can now deduce that two conditions should be satisfied to make the reacting flow effect more pronounced from a flutter point of view. First, the temperature downstream of the bow shock should be higher to increase the difference of the air equilibrium compositions before and after the shock. The farther the mixture behind the shock is from the equilibrium, the longer it will change its composition downstream, resulting in more significant K changes over the body.

Second, the flow speed behind the shock should be smaller to make visible the change in the mixture composition along the body. Indeed, if the flow speed is too high, the composition of the mixture will not change much during the motion of the flow particles along the structure, but for lower speed, this change will be more pronounced, which will result in essentially nonconstant piston theory coefficient K.

Both conditions are satisfied, and hence reacting flow effects are more pronounced for sufficiently strong (high-angle) oblique shock waves that significantly increase the temperature and reduce the velocity downstream.

III. Example

A. One-Dimensional Flow over a Wedge Behind the Shock Wave

To validate the accuracy of the piston theory with nonconstant coefficient K in reacting flow, we consider an example of a one-dimensional airflow. At the inlet, which was assumed to be the point just behind the shock (Fig. 2a), we specify Mach number $M_{\text{in}} = 2$, temperature $T_{\text{in}} = 6000$ K, pressure $p_{\text{in}} = 100{,}000$ Pa, and mixture composition consisting of 21% O_2 and 79% N_2. These parameters correspond to the flow over a wedge behind the shock wave with freestream Mach number $M_\infty = 15$, altitude 34 km, and half of the wedge angle $\alpha = 37.5$ deg. Note that the nonequilibrium flow behind the oblique shock is actually not one-dimensional and not self-similar, but this non-one-dimensionality does not affect the piston theory, which is why it is neglected in this example for the sake of clarity. The flow parameters and its composition were calculated downstream ($x > 0$) by solving Navier–Stokes equations for the mixture coupled with a system of 17 chemical reactions [14] not involving ionization, using Ansys CFX code.

Figure 3a shows the evolution of the calculated mass fractions of air components. It is seen that fractions of molecular oxygen and nitrogen drop, whereas O and NO appear (fraction of atomic nitrogen N stays negligible at this temperature). As a result of the reactions, the specific heat rate of air changes downstream (Fig. 3b); also, the flow temperature essentially drops (Fig. 3c) because a portion of the internal energy is used for the dissociation of O_2 and N_2 molecules. Because of the decrease in temperature, flow pressure also decreases (Fig. 3d), and the flow accelerates (Fig. 3e). Finally, the piston theory coefficient K [Eq. (5)] is essentially changed (Fig. 3f), being 19% lower at a distance of 2 m downstream from the inlet.

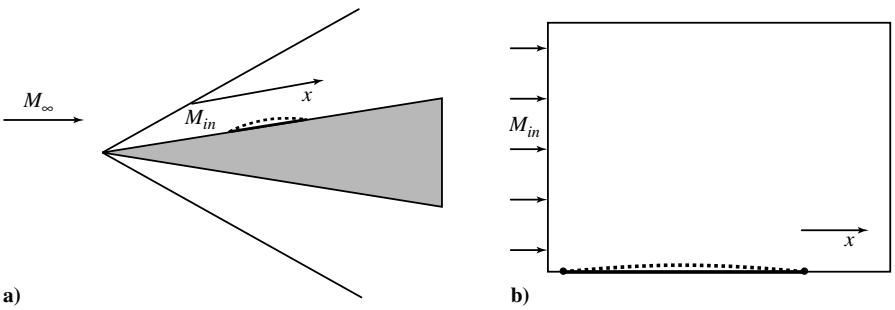

Fig. 2 Representations of a) one-dimensional flow over the unperturbed wedge surface behind the shock wave, and b) two-dimensional flow over a bent panel (leading- and trailing-edge locations are shown by circles).

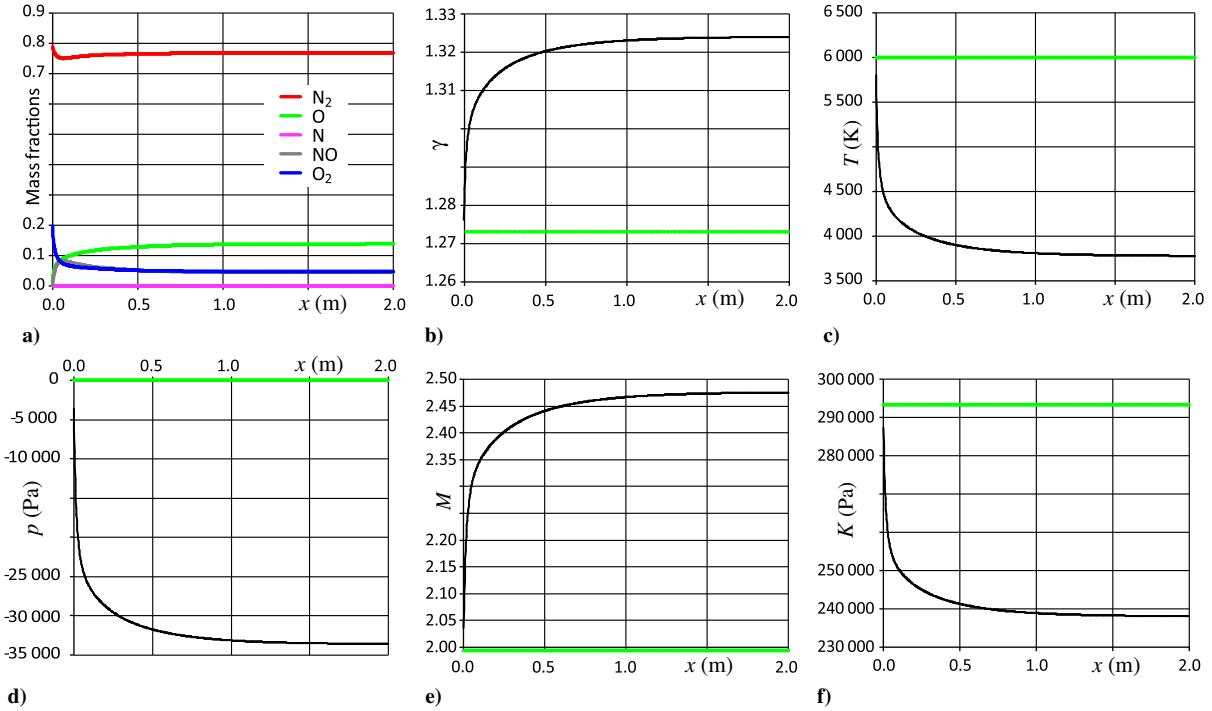

Fig. 3 Distribution of a) mass fractions, b) specific heat rate, c) temperature, d) pressure drop $p - p_{in}$, e) Mach number, and f) piston theory coefficient K for reacting (black) and nonreacting (green) flows.

B. Comparison of Unsteady Pressure with Two-Dimensional Calculation

Let us now validate the use of Ackeret's formula [Eq. (5)] for the pressure perturbation due to the wall deflection in the reacting flow. Consider a skin panel of length $L = 0.3$ m located at the surface of a wedge (Fig. 2a) that occupies the range $x_0 = 0.01 < x < 0.31$ m. Assuming simply supported boundary conditions at the panel leading and trailing edges, its first natural mode is

$$W(x) = A \sin\left(\pi(x - x_0)/L\right) \tag{6}$$

Let us consider the steady two-dimensional flow domain shown in Fig. 2b (solid and dotted bold curves show the panel in its unperturbed flat state and perturbed state given by Eq. (6)). At the inlet, we specify the same parameters as in the one-dimensional flow analysis. At the top boundary of the simulation domain, a free-slip wall condition is specified (i.e., normal velocity and shear stress are zero). At the bottom boundary, deflection of the wall in the shape given by Eq. (6) is set. In this study, we neglect the boundary-layer effect so that free-slip wall condition is specified over the deformed wall. As in the one-dimensional study, the Navier–Stokes equations with 17 reactions of the system [14] are solved using Ansys CFX. The numerical mesh consists of 122,180 finite volumes. Based on the convergence study, a maximum residual of 10^{-4} was set as the convergence criterion.

For the amplitude $A = 0.0002$ m, Fig. 4 shows the calculated pressure distribution for cases of reacting and nonreacting flows (leading- and trailing-edge are shown by circles). It is seen that, although for nonreacting flow the deviation of the flow pressure from zero is caused by the panel deflection, for the reacting flow there is also a bulk pressure decrease due to chemical reactions. Figure 5a shows the distribution of $K(x)$ along the panel from one-dimensional analysis (zoomed view of Fig. 3f); in the reacting flow, K drops by 10% at the trailing edge of the panel compared to its value at the leading edge, which should result in an increased critical Mach number. Figure 5b shows the resulting pressure perturbation for the amplitude from the two-dimensional calculations and that obtained by Eq. (5) for reacting and nonreacting flows, respectively. It is seen that, in both cases, the two-dimensional pressure perturbation and the calculations through Eq. (5) are in excellent agreement.

We conclude that, as expected, Eq. (5) can be used in the case of reacting flow, and the effect of chemical reactions essentially consists only of the nonconstant $K(x, y)$ distribution.

C. Contribution of the Perturbation of the Mixture Composition to Pressure Perturbation

To estimate the effect of the flow composition change due to surface deflection, the corresponding pressure perturbation component was calculated in the following manner. The pressure of the mixture is the sum of partial pressures produced by each of five components of the mixture:

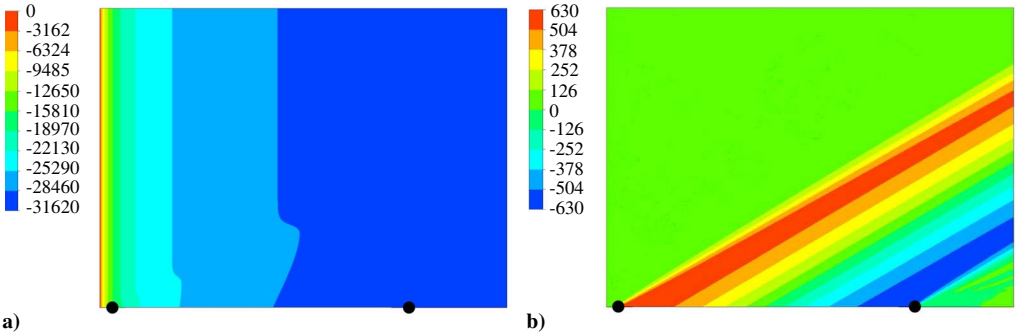

Fig. 4 Distribution of pressure $p - p_{in}$ (in pascals) over the specified panel shape for a) reacting, and b) nonreacting flows.

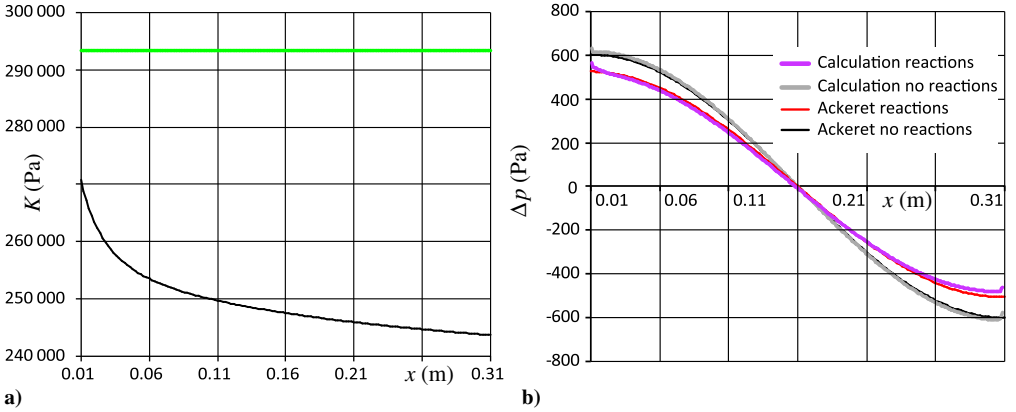

Fig. 5 Distribution of a) $K(x)$ for reacting (black) and nonreacting (green) flow over the panel, and b) pressure perturbation according to two-dimensional calculation and Eq. (5).

$$p = \sum_{i=1}^{5} \rho \kappa_i \frac{R}{m_i} T \tag{7}$$

where κ_i, and m_i are the mass fraction, and molar mass of ith component, respectively, and ρ, R and T are the density, gas constant and temperature, respectively. Then, to retain only the effect of the mixture composition change, we assume that the temperature of the mixture is unchanged because of the deflection of the surface (its change is actually taken into account by the piston theory component of the pressure perturbation), and the pressure perturbation due to composition change is given by

$$\Delta p = \sum_{i=1}^{5} \rho \Delta \kappa_i \frac{R}{m_i} T \tag{8}$$

where $\Delta\kappa_i$ are the perturbations of mass fractions due to the deflection of the surface.

The calculation result is shown in Fig. 6. It is clearly seen that the pressure perturbation generated by the mixture composition change due to surface deflection is less than 5% of the pressure perturbation due to the surface deflection calculated without considering the composition change (Fig. 5b). That is why we conclude that the change in mixture composition can be neglected when calculating the unsteady pressure.

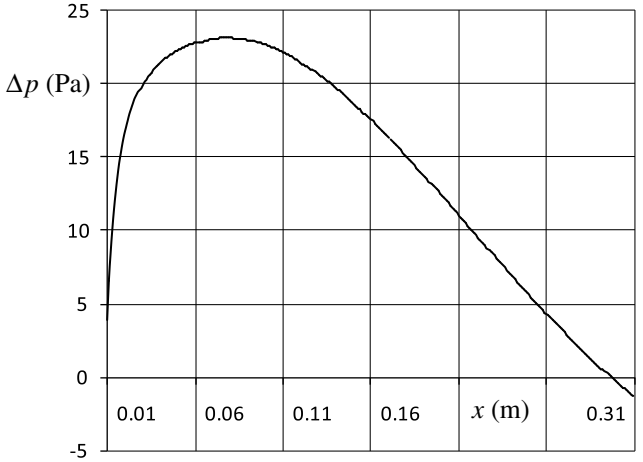

Fig. 6 Perturbation of the flow pressure due to mixture composition change caused by the surface deflection.

D. Impact of the Reacting Flow on the Flutter Boundary

Now, consider the dynamics of a skin panel (Fig. 2). Its linear equation of motion has the form

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + \rho_m h \frac{\partial^2 w}{\partial t^2} + \Delta p = 0 \tag{9}$$

where D and h are its bending stiffness and thickness, respectively, and ρ_m is the panel material density. Substitution of the quasi-static expression given by Eq. (5) yields the coupled aeroelastic equation of motion:

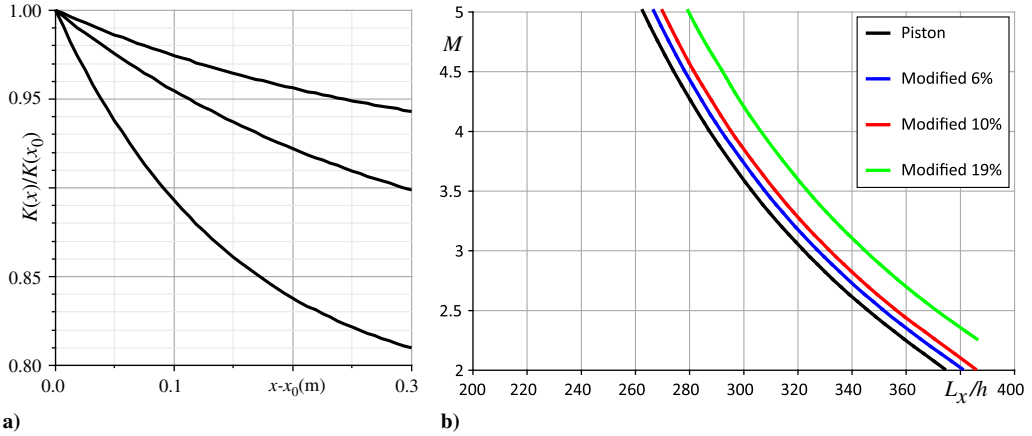

Fig. 7 Representations of a) modeled distributions $K(x)$, and b) panel flutter boundary in the $L_x/h - M$ plane, with nonreacting flow and modifications of the piston theory coefficient.

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + \rho_m h \frac{\partial^2 w}{\partial t^2} + K(x)\frac{\partial w}{\partial x} = 0 \quad (10)$$

Its stability is analyzed numerically by the Bubnov–Galerkin method. The computational code and its validation are described in [12] and are not discussed here for the sake of brevity.

To analyze the effect of nonconstant piston theory coefficient $K(x)$ on the flutter boundary, we have considered a simply supported rectangular steel panel of thickness $h = 1$ mm, width $L_y = 0.3$ m (spanwise), and different lengths L_x (chordwise) and calculated stability boundary for Eq. (10). The following $K(x)$ distributions are considered (Fig. 7a), modeling different drop amounts compared to the inlet value: 1) $K(x) = K(x_0)(1 + 0.22(\exp(-2(x - x_0)/L_x) - 1))$, which models the drop of K by 19%; 2) $K(x) = K(x_0)(1 + 0.16(\exp(-(x - x_0)/L_x) - 1))$, which models the drop of K by 10%; and 3) $K(x) = K(x_0)(1 + 0.09(\exp(-(x - x_0)/L_x) - 1))$, which models the drop of K by 6%.

Calculated flutter boundaries are shown in Fig. 7b. It is seen that, for a panel length of 0.3 m, the drop of K by 10 % (as, for example, in Fig. 5a) results in a change in critical Mach number M_{cr} by ≈0.2; a higher drop of K, by 19%, changes M_{cr} by 0.6. For shorter panels, the effect of the same amount of the drop of K becomes larger.

IV. Conclusions

It has been shown that nonequilibrium chemical reactions in hypersonic flow and the corresponding change in local specific heat rate and temperature along the body surface results in a nonconstant, decreasing downstream piston theory coefficient, whereas the piston theory itself stays valid. This results in an increase in critical Mach number M_{cr} compared to nonreacting flow. Considered examples show that, depending on specific flight conditions, this can yield an increase in M_{cr} by the order of 0.2 for a panel of 0.3 m length, which is of the order of 10% of M_{cr} calculated without taking reactions into account.

The effect of the change in air composition because of the surface deflection on the pressure perturbation is an order lower than the pressure perturbation calculated without taking the composition change into account. In other words, the perturbation of the mixture composition can be neglected in most cases.

In this study, catalytic processes on the body surface were not considered; however, their effect can be estimated in a similar manner. The results obtained can be useful in the design of lightweight and robust hypersonic flight vehicles.

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