ISSN 1028-3358, Doklady Physics, 2021, Vol. 66, No. 4, pp. 106–109. © Pleiades Publishing, Ltd., 2021.
Russian Text © The Author(s), 2021, published in Doklady Rossiiskoi Akademii Nauk. Fizika, Tekhnicheskie Nauki, 2021, Vol. 497, pp. 44–48.

MECHANICS

Experimental Validation of Linear-Stability Theory Applied to a Submerged Jet

J. S. Zayko[a,*], L. R. Gareev[a], A. D. Chicherina[a], V. V. Trifonov[a], V. V. Vedeneev[a,b], and A. I. Reshmin[a]

Presented by Academician A.G. Kulikovskii January 31, 2021

Received February 7, 2021; revised February 7, 2021; accepted February 11, 2021

Abstract—A submerged air jet of circular cross section of 0.12 m in diameter and a long laminar region (~5 jet diameters) is obtained experimentally at the Reynolds number of 5400. Within the linear analysis of stability, two branches of growing perturbations are found, which are generated by three inflection points on the experimental jet profiles. The frequency ranges of perturbations, as well as their growth rates and wavelengths are obtained. Experiments on introducing controlled perturbations into the jet with a long laminar portion are conducted. The characteristics of waves amplified in experiments due to the introduction of perturbations prove to be close to the predictions of the linear theory of stability. Thus, the applicability of the linear stability theory to the submerged jet is validated experimentally.

Keywords: linear stability theory, Rayleigh equation, submerged jet

DOI: 10.1134/S1028335821040054

The development of small perturbations in accordance with the linear stability theory was confirmed experimentally only for certain flows, for example, for the Blasius boundary layer [1, 2], the plane Poiseuille flow [3, 4], and the Poiseuille flow in a pipe of circular cross section [5]. Such experiments for submerged jets are complicated by the fact that, under normal conditions, the critical Reynolds numbers Re_{cr} of the transition to turbulence in jets are low; in the case of a jet with a circular cross section, Re_{cr} ranges from 14 to 44 [6–8]. The results of the linear stability theory applied to the submerged jets were verified experimentally only for turbulent jets (in which the length of the portion before the transition amounted to 1–2 diameters), and the perturbations introduced into the jets were intense and significantly affected the flow [9–11].

At the Research Institute of Mechanics, Moscow State University, a device was designed for forming a submerged jet with the diameter of $D = 0.12$ m with the laminar portion of ~$5D$ in length at the Reynolds numbers calculated from the diameter and the average velocity in the range of 5000–10 000 [12, 13]. Such a jet enables us to conduct detailed experimental observations of the development of perturbations and compare them with the predictions of the linear stability theory applied to the velocity profiles of the jet at different distances from its beginning. In this study, we show that the experimentally obtained ranges of frequencies, wavelengths, and growth rates of the fastest growing perturbations agree with the predictions of the linear theory.

LONG LAMINAR JET

The device and the jet formed by it with a long laminar portion are described in detail in [12, 13]. Further in this study, we carry out theoretical analysis and experiments for a jet with the Reynolds number of 5400 calculated from the diameter D and the average velocity (the velocity on the jet axis is $U_c = 1.5$ m/s). The velocity profiles at different distances from the beginning of the jet were measured with a hot-wire anemometer, compared with the profiles obtained in the calculation of stationary laminar flow (see [13]), and approximated by analytical functions (Fig. 1).

THEORETICAL ANALYSIS OF STABILITY

We carried out the theoretical linear analysis of the jet stability on the basis of the inviscid approximation using the Rayleigh equation. Its basis was a reasonably high Reynolds number (5400) of the flow under consideration. An axisymmetric mode was considered.

[a] *Institute of Mechanics, Moscow State University, Moscow, 119192 Russia*
[b] *Faculty of Mechanics and Mathematics, Moscow State University, Moscow, 119991 Russia*
**e-mail: zayko@imec.msu.ru*

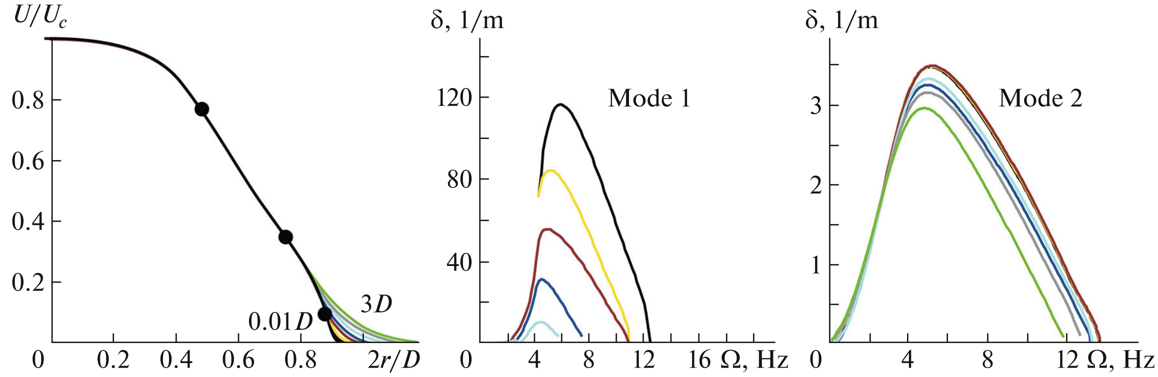

Fig. 1. Velocity profiles nondimensionalized to the maximal velocity and the jet radius at various distances from the jet beginning (the circles show the generalized inflection points of the initial profile); the growth rates versus frequencies for the modes of two branches of growing perturbations. The colors correspond to those of the velocity profiles and indicate different distances from the jet beginning.

We analyzed nine longitudinal velocity profiles at different distances from the beginning of the jet, the velocity profile $U(r)$ in the initial cross section of the jet has three generalized inflection points (the points where $(U'(r)/r)' = 0$, the prime denotes derivative with respect to r) (see Fig. 1). It was found that there are two branches of growing perturbations in the flow under study. It should be noted that there is only one branch of growing perturbations generated by one generalized inflection point in the classical velocity profiles (top-hat shock, self-similar far-downstream). In the jet under consideration in this study, the first branch of growing perturbations is generated by the inflection points located near the boundary, and the second branch is generated by the inflection point located closer to the jet axis (see Fig. 1). The rates of spatial growth and the wavelengths of growing perturbations are obtained. It turns out that the most rapidly growing perturbations of the first branch have frequencies in the range of ~4–8 Hz (Fig. 1). In this case, when moving downstream, the outer inflection point shifts into a jet due to a change in the profile caused by its viscous "spreading," which is accompanied by a decrease in the highest growth rate of the perturbation and a narrowing of the frequency range of growing waves. The frequencies of the fastest growing perturbations of the second branch also lie in the range of ~4–8 Hz (Fig. 1); in the laminar-jet region, this mode undergoes no significant changes associated with the evolution of the velocity profile.

EXPERIMENT ON INTRODUCING CONTROLLED PERTURBATIONS INTO THE JET

A device for introducing perturbations into the jet is shown in Fig. 2. It consists of thin metal strings stretched between drives on which a ring of thicker wire is fixed coaxially to the jet (the cross sections of the strings and the wire were selected so that the Reynolds numbers of the flow around them were lower than the Reynolds numbers for forming the Karman vortex stream behind them). The strings with a ring are actuated by a frequency- and amplitude-controlled drive, which enables us to move the ring along the jet axis with a set frequency and amplitude. Rings of two diameters were used: ring no. 1 has the diameter located between two inflection points generating the first branch of the growing perturbations; the radius of ring no. 2 is identical to the distance from the jet axis to the inflection point located closer to the jet axis and generating the second branch of growing perturbations.

It was verified that the installation of a motionless ring in the jet does not significantly affect the flow, and the laminar-portion length of the jet does not decrease; see Fig. 3, which shows a photograph of the jet with the ring at rest (0 Hz). Two series of experiments were conducted: in the first series, the goal was the excitation of the first mode obtained theoretically (ring no. 1 was used); in the second series, it was the excitation of the second mode generated by the inflection point near the jet axis (ring no. 2 was used). In the first series of experiments, the jet was sown with glycerin particles, and the jet was surveyed in the section of the laser sheet under vibrations of ring no. 1 with frequencies from 0 to 12 Hz and a step of 0.25 Hz (see photographs of the jets in Fig. 3). The flow patterns were obtained at different frequencies of the ring vibrations and wavelengths formed at the jet boundary. In the second series of experiments, the signals were synchronously recorded from the rangefinder, which detected the space position of ring no. 2, and the hot-wire anemometer sensor, which detected the readings along the diameter of the jet at different distances from its beginning. The correlation patterns of the signals detected were plotted for different frequencies, which enabled us to determine the wavelengths having a set frequency.

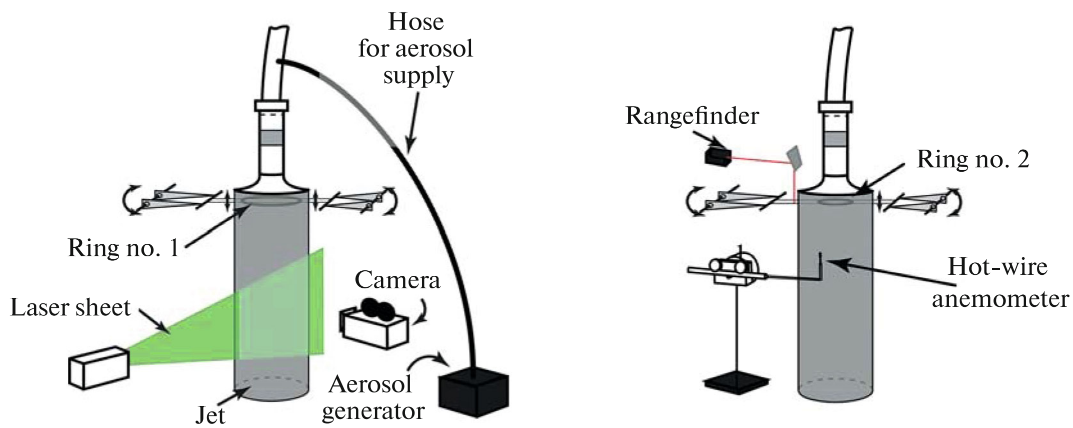

Fig. 2. Schematic representation of flow, the introduction of perturbations into the jet, the laser sheet, and the hot-wire anemometer with a rangefinder.

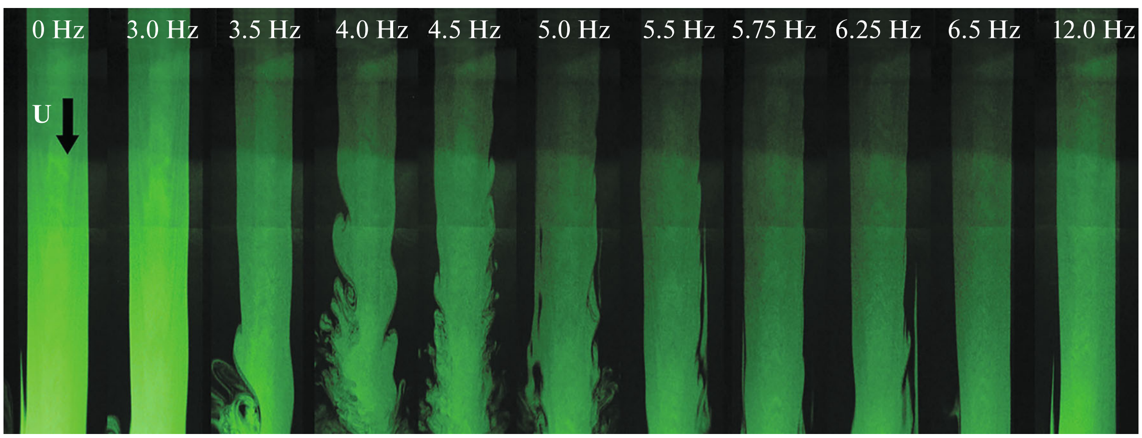

Fig. 3. Photographs of the jet in the section of the laser sheet at different vibration frequencies of ring no. 1.

COMPARISON OF EXPERIMENTAL DATA WITH PREDICTIONS OF THE LINEAR STABILITY THEORY

Figure 3 shows the jet in the section of the laser sheet at different vibration frequencies of ring no. 1 (and the same vibration speed). It can be seen that there is no significant effect of vibrations on the flow at frequencies below 3 Hz and above 6.5 Hz. At the frequencies in the range of 3.5–6.25 Hz, a reduction in the laminar section of the jet is observed. These frequencies correspond to the theoretical range of frequencies of the fastest growing perturbations. Based on the vizualisation results, the wavelengths arising at the boundary of the jet were determined. They agree with the wavelengths of the first branch of growing perturbations obtained theoretically (see the comparison in the graph in Fig. 4).

The wavelengths found from the correlation patterns for different vibration frequencies of ring no. 2 are in good agreement with the wavelengths of the second branch of growing perturbations (see the graph in Fig. 4, triangle symbols).

In Fig. 4, we also show the comparison of the theoretically obtained growth of the first-branch perturbation $\int \delta(x)dx$ for a frequency of 5 Hz (taking into account the evolution of the downstream jet-velocity profile) with the amplitude of the velocity fluctuations from an experiment in which ring no. 1 generating the first branch of growing perturbations oscillated with a frequency of 5 Hz (the signal of the hot-wire anemometer is filtered around the frequency of 5 Hz). In the immediate vicinity of the vibrating ring, we observe the whole spectrum of waves, which rapidly decay, instead of only the presence a perturbed mode after which the linear (in logarithmic scale) growth of the eigenmode is observed (the fluctuations grow exponentially downstream), which is replaced by the stage of nonlinear development at a certain distance. Despite the shifts between the theoretical plot and the two series of experimental points in the axial direction

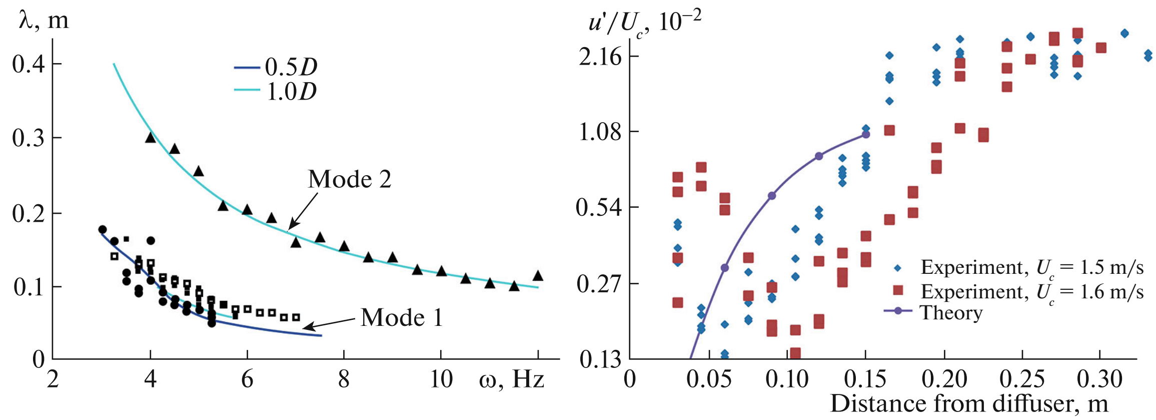

Fig. 4. Comparison between the experimental and theoretical dependences of the wavelengths λ on frequency. The symbols show the wavelengths obtained in experiments, and the curves represent the wavelengths obtained theoretically. Comparison of the theoretically established growth rate of perturbation with the frequency of 5 Hz and the highest amplitude of the relative velocity fluctuations u'/U_c on the distance from the beginning of the jet.

(caused by different measurement conditions), there is agreement in the growth rate between the theory and the experiment.

CONCLUSIONS

It is shown experimentally in this study that the linear inviscid stability theory correctly predicts the properties of waves in the laminar submerged jet of circular cross section at the Reynolds number of 5400. In the experiments, the waves corresponding to the theoretically obtained eigenmodes of both branches of growing perturbations were amplified. The wavelengths, the frequency ranges, and the perturbation-growth rate established experimentally are close to the corresponding values predicted in the framework of the linear inviscid stability theory. Thus, due to the use of a jet of large diameter ($D = 0.12$ m) with a long laminar portion, which makes it possible to study in detail the development of waves, the inviscid linear stability theory was verified experimentally in this study as applied to a laminar submerged jet.

FUNDING

This study was supported by the Russian Science Foundation, project no. 20-19-00404.

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Translated by V. Bukhanov