ISSN 1052-6188, Journal of Machinery Manufacture and Reliability, 2021, Vol. 50, No. 4, pp. 342–350. © Allerton Press, Inc., 2021.
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NEW TECHNOLOGIES IN MECHANICAL ENGINEERING

Study of Aeroelastic Phenomena of the Hull and Thin-Walled Structures of Unmanned Aircraft at High Supersonic Speeds

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Received June 11, 2020; revised February 17, 2021; accepted February 24, 2021

Abstract—Flutter of unmanned aircraft at hypersonic speeds is one of the understudied problems facing the designers of hypersonic vehicles. Modern methods for calculating aeroelastic stability either solve simplified versions of real problems (for example, without taking physicochemical phenomena into account) or require high computing power. This paper describes a methodology for calculating the supersonic and hypersonic flutter of an aircraft using standard engineering software and additionally developed software modules. The justification of the need to refine the existing methods for calculating the aircraft aeroelasticity taking into account the real geometry of the structure and with the possibility of accounting for physicochemical processes occurring in the air during the movement of bodies at high speed is provided. The theoretical principles of calculating aeroelastic stability taking into account these factors are developed, and three examples of calculating the aeroelastic stability of model objects are given.

Keywords: hypersonic aerodynamics, aeroelasticity, flutter, flow, CFD

DOI: 10.3103/S105261882103002X

The flight of aerial vehicles at high speeds, including supersonic and hypersonic speeds, may be accompanied by the phenomenon of flutter of various structural elements of the aircraft: wings, rudders, hull, and skin panels [1]. Current engineering methods for calculation of aeroelastic stability often use simplified approaches: rudders and hulls are idealized as plates and cylindrical shells that are flown over with a zero angle of attack. In reality, the geometry of the outer contours of the body is usually more complex, and states of flight include nonzero angles of attack, but these features are not taken into account in the flutter calculations. A specific feature of hypersonic flow is that the air after the bow shock at these velocities is in a nonequilibrium state and the processes of dissociation and recombination of molecules accompanied by various chemical reactions become a significant factor affecting the flow. Under such conditions, the flutter resistance limit should be determined with allowance for chemical reactions and kinetic processes in the incoming air flow. Until now, these effects have remained almost unexplored and have not been taken into account in the practice of engineering calculations.

At the same time, both Russian and foreign specialists design aerial vehicles with flight Mach numbers $M = 6–15$, the outer contours of which have a complex shape, and the flow of which may be accompanied by the abovementioned chemical transformations. To achieve high flight speeds, the design of such vehicles must have a high weight perfection, which will ensure the placement of the onboard equipment and warhead payload necessary for solving combat missions on hypersonic aircraft (HSA). The HSA structures can have thin-walled skin panels and elastic empennage; therefore, they can be susceptible to flutter and possible flutter-induced damage. Calculations of the flutter resistance limits of such vehicles based on simplified methods can give quantitatively incorrect predictions of the stability limits [2]. Thus, it is highly important to develop a more advanced method for calculating aeroelastic stability that would take into account both the real geometry of the outer contours of the body and chemical transformations in air and to create software modules and libraries on its basis for calculating flutter in a supersonic flow.

METHOD FOR THE CALCULATION OF AEROELASTIC STABILITY

Let us derive the system of equations of motion of an aeroelastic system in generalized coordinates from the system of Lagrange's differential equations of motion [3]

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{u}_i}\right) - \frac{\partial L}{\partial u_i} = Q_i^n,$$

where L is the Lagrangian equal to the difference between the kinetic and potential energies, Q_i^n denotes nonpotential generalized forces caused by aerodynamic action, and u_i denotes the generalized coordinates for which we will take the amplitudes of the expansion of the surface displacement vector of the elastic body in terms of vibration eigenmodes (i.e., in fact, the Bubnov–Galerkin method is used).

To obtain the expressions for generalized forces, let us consider the elementary work δW performed by aerodynamic pressure on an elementary increment of generalized coordinates δu_j:

$$\delta W = \sum_{j=1}^{N} \int_S \vec{p}_n \left(\sum_{i=1}^{N} u_i(t) \cdot \vec{w}_i(x,y,z) \right) \cdot \vec{w}_j \delta u_j d\sigma$$

$$= \sum_{i,j=1}^{N} \int_S \vec{p}_n \left(u_i(t) \cdot \vec{w}_i(x,y,z) \right) \cdot \vec{w}_j(x,y,z) \delta u_j d\sigma,$$

where $\vec{p}_n$ is the increment of the pressure vector acting on the body (viscous stresses in air are neglected), S is the surface area of the body, $\vec{w}_i(x,y,z)$ is the distribution of the displacements according in the ith eigenmode, $\vec{w}_j \delta u_j$ is the elementary movement of the body with variation of the jth generalized coordinate, and N is the number of eigenmodes considered.

It follows from the d'Alembert–Lagrange variational principle [3] that

$$Q_j = \frac{\partial(\delta W)}{\partial(\delta u_j)} = \sum_{i=1}^{N} \int_S \vec{p}_n \left(u_i(t) \cdot \vec{w}_i(x,y,z) \right) \cdot \vec{w}_j(x,y,z) d\sigma = \sum_{i=1}^{N} q_{ji} u_i(t).$$

Thus, the column vector of linearized generalized aerodynamic forces can be represented as the product of the matrix

$$\hat{K}_a = (q_{ij}) = \int_S P\left(\vec{w}_j(x,y,z)\right) \cdot \vec{n}(x,y,z) \cdot \vec{w}_i(x,y,z) d\sigma,$$

called the aerodynamic stiffness matrix, and the column vector of generalized coordinates. Here, P is pressure perturbation caused by deformation of the body according to its jth eigenmode and $\vec{n}$ is the normal to the body surface oriented inward into the body. Since this study considers high supersonic and hypersonic velocities and relatively small objects, the characteristic time of motion of a gas particle along the body is an order of magnitude less than the characteristic periods of vibrations of the body in eigenmodes that are feasible to take into account in flutter calculations (small Strouhal numbers). Therefore, the flow can be considered quasi-stationary, and the pressure distribution depends only on the displacement, but not on the velocity of the body surface. Thus, the aerodynamic damping matrix is neglected, which gives slightly lower critical velocities and goes in reserve.

The expressions for the kinetic T and potential U energy of a linearly elastic body in generalized coordinates have the form

$$T = \frac{1}{2}(\dot{u}_1 \cdots \dot{u}_N)\hat{M}\begin{pmatrix}\dot{u}_1 \\ \vdots \\ \dot{u}_N\end{pmatrix}, \quad U = \frac{1}{2}(u_1 \cdots u_N)\hat{K}\begin{pmatrix}u_1 \\ \vdots \\ u_N\end{pmatrix},$$

which gives an expression for the Lagrangian $L = T - U$. In these expressions, matrix $\hat{M}$ is the mass matrix and matrix $\hat{K}$ is the matrix of structural stiffness. The Lagrange equations of motion of the body in the matrix form will appear as follows:

$$\hat{M}\ddot{u} + \hat{D}\dot{u} + \hat{K}u = \hat{K}_a u \rightarrow \hat{M}\ddot{u} + \hat{D}\dot{u} + \left(\hat{K} - \hat{K}_a\right)u = 0.$$

They also take into account the structural damping matrix $\hat{D}$, which can be set given known values of the damping coefficients for each natural vibration.

The aeroelastic stability in the examples below was analyzed for the unloaded state of the structure (zero angle of attack and zero angle of rudder deflection) by virtue of considering the motion of fairly rigid bodies of relatively small dimensions; this, however, does not limit the generality of the method. The loads acting on the structure from the air flow are calculated using the numerical solution of the fluid dynamics equations (CFD). The system of the Navier–Stokes equations and energy equations for a viscous heat-conducting gas is solved.

The continuity equation is $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho U) = 0$.

The Navier–Stokes equations are $\frac{\partial (\rho U)}{\partial t} + \nabla \cdot (\rho U \otimes U) = -\nabla p + \nabla \cdot \tau$, where $(U \otimes U)_{ij} = \left(UU^T\right)_{ij} = U_i U_j$, the components of tensor τ are expressed as $\tau_{ij} = \mu\left(\nabla^j U_i + \nabla^i U_j - \frac{2}{3}\delta_{ij}(\nabla \cdot U)\right)$, and δ_{ij} is the Kronecker delta.

The energy equation written in terms of the total enthalpy is given by

$$\frac{\partial (\rho h_{\text{tot}})}{\partial t} - \frac{\partial p}{\partial t} + \nabla \cdot (\rho U h_{\text{tot}}) = \nabla \cdot (\lambda \nabla T) + \nabla \cdot (U \cdot \tau).$$

The system of equations is solved by the control volume method. The characteristics of a steady flow are obtained by the relaxation method. In the case when chemical reactions in the flow can be neglected (the Mach number does not exceed five), the system of equations is closed by the equation of state of a perfect gas. If chemical reactions in air at hypersonic flight speeds are taken into account, this system of equations is considered for a mixture of reacting components and is supplemented by a system of equations for chemical reactions [4].

Upon obtaining the equations of motion of the aeroelastic system in modal coordinates and calculating matrix $\hat{K}_a$, we solve the complex eigenvalue problem. Representing u as $u = Ce^{\lambda t}$, we find

$$\det\left(\hat{M}\lambda^2 + \hat{D}\lambda + \left(\hat{K} - \hat{K}_a\right)\right) = 0.$$

As a result, we obtain a set of N eigenvalues λ. The stability criterion is the condition $\operatorname{Re}\lambda \leq 0$. In the case of a real positive eigenvalue, there is divergence; in the case of a complex eigenvalue with a positive real part, there is flutter.

PRACTICAL IMPLEMENTATION OF THE METHOD

This paper describes a method for calculating flutter, in which generalized aerodynamic forces are calculated on the basis of the numerical simulation (CFD) of the flow of the real geometry of the structure with allowance for the chemical reactions occurring in the air. As shown in the previous section, due to the high flight speeds and relatively small sizes of the objects, it is assumed that the flow over the oscillating structure is quasi-stationary. The specific implementation of the method is based on the use of two commercial software packages used in engineering practice: MSC.Nastran [5, 6] for the calculation of eigenfrequencies and eigenmodes of a structure in a vacuum and solving the eigenvalue problem for a coupled aeroelastic system and Ansys CFX [7] for solving the problem of quasi-stationary flow over a structure. Additional software modules have been developed for integrating these two software packages, calculating generalized aerodynamic forces, building an aerodynamic stiffness matrix, and processing the results.

The block diagram of the calculations is shown in Fig. 1. The vibration eigenmodes and eigenfrequencies (both hull and rudder surfaces) are calculated in MSC.Nastran using the standard finite element method (SOL 103 solver) and are transferred to the Ansys CFX fluid dynamic package by the developed software module. The package calculates the flow over the structure upon deformation of the hull in eigenmodes. Further, the generalized aerodynamic forces, which form the aerodynamic stiffness matrix, are calculated using the second module developed. This matrix is transferred to MSC.Nastran for calculating the complex eigenfrequencies of vibrations of a coupled aeroelastic system (SOL 110 solver), which is carried out using the Bubnov–Galerkin method (in Nastran terminology, in modal coordinates), which requires a relatively small number of eigenmodes taken into account in the computation of generalized

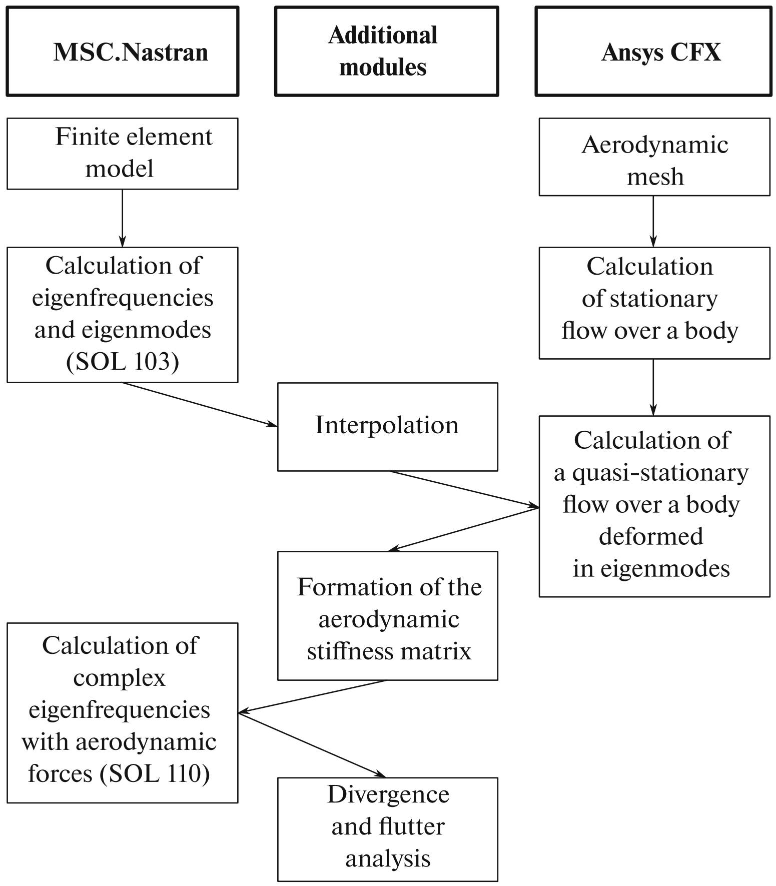

Fig. 1. Block diagram of the calculations.

aerodynamic forces. According to the theory, the criterion of instability (divergence or flutter) is the presence of an eigenfrequency with a positive real part.

APPLICATION EXAMPLES

Three objects were investigated using the method developed, and the results were compared with the data of field experiments.

Figures 2a and 2b show the geometry of the first object under study, an unmanned maneuverable aircraft. All calculations for this model were carried out for flight parameters at an altitude of 25 km and Mach number $M = 5$. The description of the lowest vibration eigenmodes of the model is given in Table 1 and in Fig. 3. The calculations took into account two hull eigenmodes (the first and second bending mode with numbers 8 and 10), as well as three eigenmodes of the rudders (bending, bending-torsional, and torsional modes, numbers 20, 28, and 30). It should be noted that in Table 1 several frequencies correspond to one mode, since the corresponding eigenmode occurs on different rudders, and the small difference in frequency values is due to the asymmetric arrangement of the onboard equipment and payload.

The calculation of complex eigenvalues in the SOL 110 MSC.Nastran module showed the absence of flutter (Table 2).

This is evidenced by zero values of aerodynamic damping $\Delta = -\mathrm{Re}(\lambda)$ given for all calculated physical vibration frequencies $\omega = \dfrac{\mathrm{Im}(\lambda)}{2\pi}$, which are almost the same as the vibration frequencies in a vacuum. This is due to the rather high eigenfrequencies of the rudders and the high flight altitude in the regime considered (25 km), at which the aerodynamic effect of the flow on the rocket is small. The calculation results are confirmed by field tests.

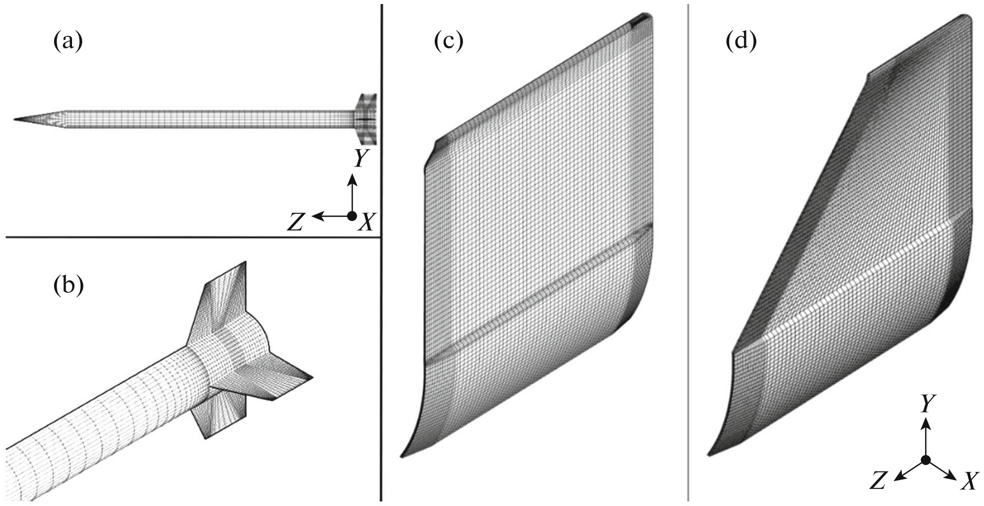

Fig. 2. Geometry of three computational models: (a), (b) first; (c) second; (d) third.

Figures 2c and 2d show the geometry of the second and third objects under study, the empennage of a maneuverable aircraft. According to the experimental data, the use of a trapezoidal rudder (Fig. 2d) caused no flutter in various design flight states. At the same time, replacing the trapezoidal empennage

Table 1. Eigenmodes: First model

Mode	Frequency, Hz	Eigenmode type	Mode	Frequency, Hz	Eigenmode type
1	0	Loose move	16	179.19	Rudder, bending 1
2	0	Loose move	17	185.07	Rudder, bending 1
3	0	Loose move	18	197.42	Rudder, bending 1
4	0	Loose move	19	201.39	Rudder, bending 1
5	0	Loose move	20	201.80	**Rudder, bending 1**
6	0	Loose move	21	285.09	Hull, longitudinal
7	38.98	Hull, bending 1	22	325.78	Hull, bending 3
8	38.98	**Hull, bending 1**	23	325.78	Hull, bending 3
9	97.79	Hull, bending 2	24	400.03	Hull, mixed
10	97.90	**Hull, bending 2**	25	400.03	Hull, mixed
11	117.16	Rudder, bending in the rudder plane	26	444.81	Hull, mixed
12	122.77	Rudder, bending in the rudder plane	27	444.81	Hull, mixed
13	123.16	Rudder, bending in the rudder plane	28	444.93	**Rudder, torsion 1**
14	124.02	Rudder, bending in the rudder plane	29	483.48	Rudder, torsion 2
15	179.14	Rudder, bending 1	30	483.64	**Rudder, torsion 2**

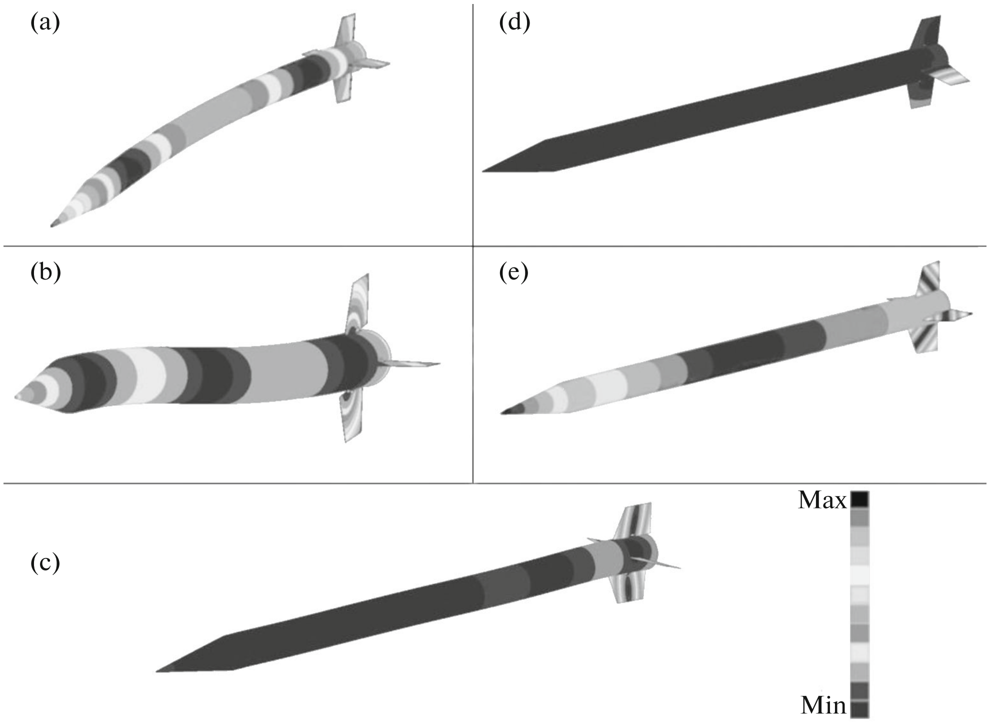

Fig. 3. Vibration eigenmodes of the first model. The scale is increased 15 times: (a) mode 8; (b) mode 10; (c) mode 20; (d) mode 28; (e) mode 30.

geometry with a rectangular one (Fig. 2c) led to flutter. The calculations based on the method proposed in this paper confirm the experimental results. All the calculations were carried out with parameters corresponding to zero altitude above sea level.

The obtained vibration eigenmodes of the second model in a vacuum are shown in Fig. 4 and described in Table 3.

Table 2. Results of calculating eigenvalues: first model. $M = 5$; $H = 25$ km

Mode	Δ	ω, Hz	Mode	Δ	ω, Hz
1	0	0	16	0	179.19
2	0	0	17	0	185.07
3	0	0	18	0	197.42
4	0	0	19	0	201.39
5	0	0	20	0	201.83
6	0	0	21	0	285.09
7	0	38.97	22	0	325.78
8	0	38.98	23	0	325.78
9	0	97.79	24	0	400.03
10	0	97.90	25	0	400.03
11	0	117.16	26	0	444.81
12	0	122.77	27	0	444.81
13	0	123.16	28	0	444.95
14	0	124.02	29	0	483.48
15	0	179.14	30	0	483.67

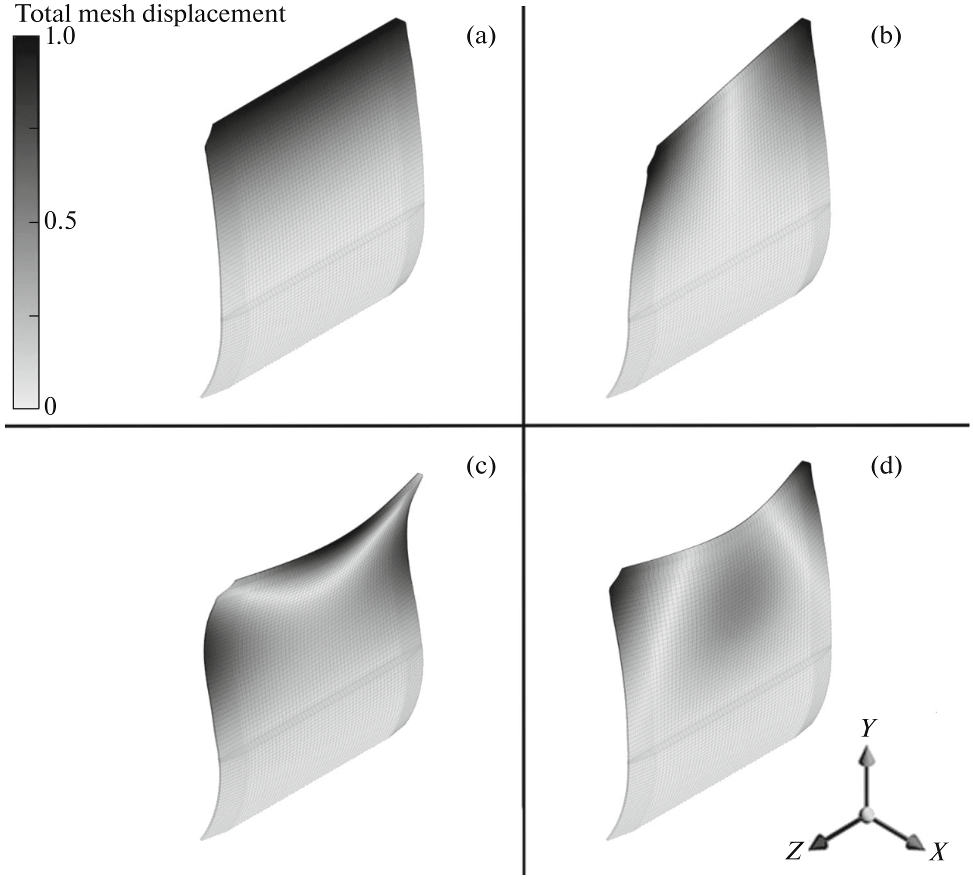

Fig. 4. Vibration eigenmodes of the second model. The scale is increased 15 times: (a)–(d) modes 1 through 4, respectively.

Table 4 shows the results obtained from the calculations based on the method for the rectangular empennage. The calculation was first carried out for the Mach number $M = 2.5$, and it was concluded that a coupled flutter in the second mode of vibration was present in this flight state. Further, the calculations were carried out for $M = 2$ and $M = 1.5$ (note that, in the latter case, the assumption that the flow is quasi-stationary is obviously wrong, and this calculation is of purely academic interest). The values obtained of the aerodynamic damping coefficients also indicate the presence of the same type of flutter.

Similar calculations were carried out for the trapezoidal empennage model. The obtained eigenmodes of the model are shown in Fig. 5 and described in Table 5. The calculation first was carried out for $M = 2$ and then for $M = 6$ (Table 6). The Δ value remained equal to zero, which indicates the absence of flutter.

CONCLUSIONS

This paper presents the theory and methodology for calculating the flutter of aircraft structures at high flight velocities taking into account the real geometry of the structure and the possibility of accounting for

Table 3. Eigenmodes: second model

Mode	Frequency, Hz	Eigenmode type
1	87.40	Bending 1
2	156.97	Torsional 1
3	376.72	Bending 2
4	436.55	Plate-like

Table 4. Results of calculating complex eigenvalues: second model. $H = 0$ km. Left to right: $M = 2.5$; $M = 2$; $M = 1.5$

Mode	Δ, Hz	ω, Hz	Mode	Δ, Hz	ω, Hz	Mode	Δ, Hz	ω, Hz
1	308	172.55	1	244	145.22	1	171	114.0
2	−308	172.55	2	−244	145.22	2	−171	114.0
3	0	375.51	3	0	386.35	3	0	378.1
4	0	432.07	4	0	424.23	4	0	408.6

Table 5. Eigenmodes: Third model

Mode	Frequency, Hz	Eigenmode type
1	145.20	Bending 1
2	394.85	Torsional 1
3	585.12	Bending 2
4	887.48	Torsional 2

the features of hypersonic flow (chemical transformations, viscous effects, etc.). The practical implementation of the calculation method with the use of the software modules and partial use of standard engineering software is described. Examples of calculating the flutter of model aircraft structures are given; their results are consistent with the experiments.

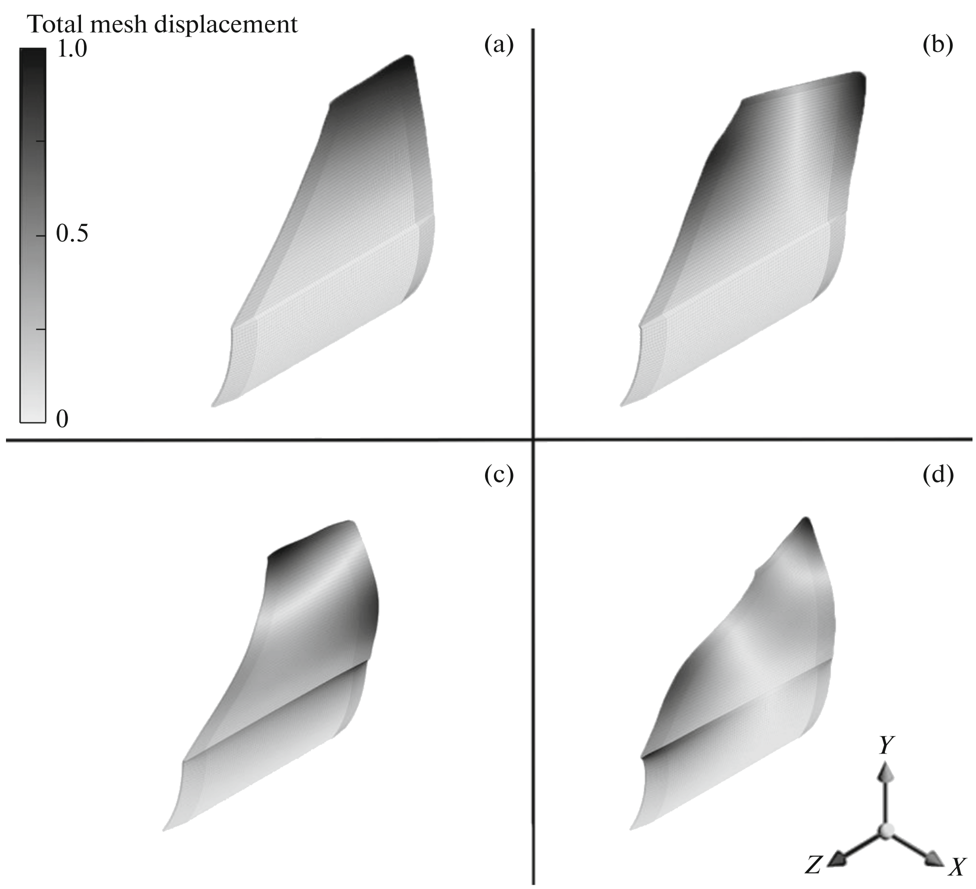

Fig. 5. Vibration eigenmodes of the third model. The scale is increased 15 times: (a)–(d) modes 1 through 4, respectively.

Table 6. Results of calculating complex eigenvalues: third model. $H = 0$ km. Left to right: $M = 2$; $M = 6$

Mode	Δ	ω, Hz	Mode	Δ	ω, Hz
1	0	168.02	1	0	254.38
2	0	389.10	2	0	409.99
3	0	593.83	3	0	606.13
4	0	880.11	4	0	888.44

This method can be applied to study the flutter of other aircraft, as well as their structural components. The method can be used in conjunction with other software systems, including Russian ones, by developing additional modules that provide connections between the elasticity and fluid dynamics solvers and performing intermediate calculations.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

FUNDING

The paper was prepared under partial financial support of the Ministry of Science and Higher Education of the Russian Federation within the Program of "Supersonic" (agreement № 075-15-2020-923).

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Translated by M. Chubarova